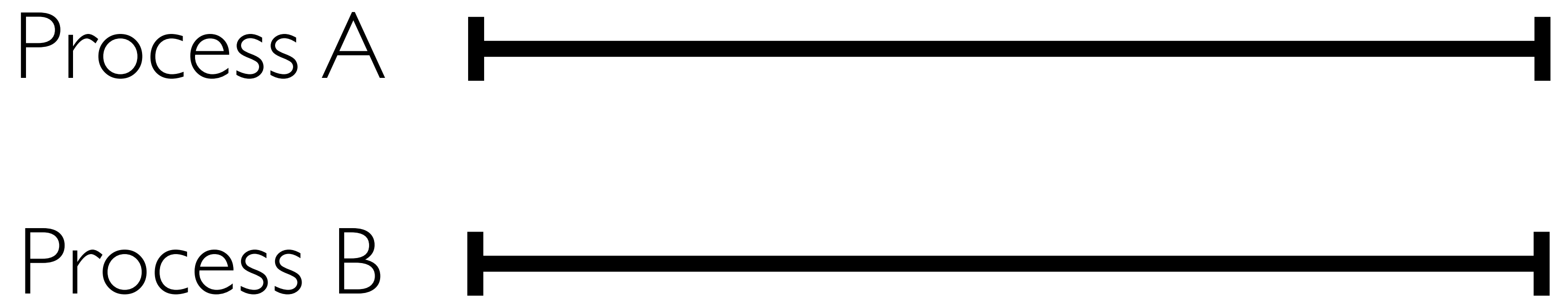
Logical relations for fine-grained concurrency

Aaron Turon

with

Jacob Thamsborg, Amal Ahmed,
Lars Birkedal, Derek Dreyer

Concurrency is overlapped execution:



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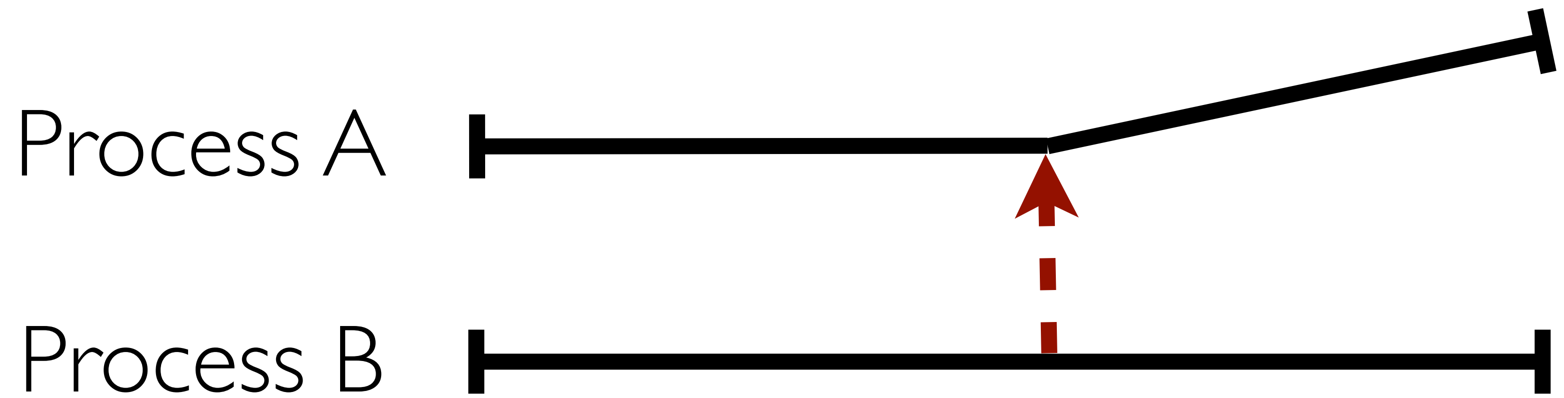

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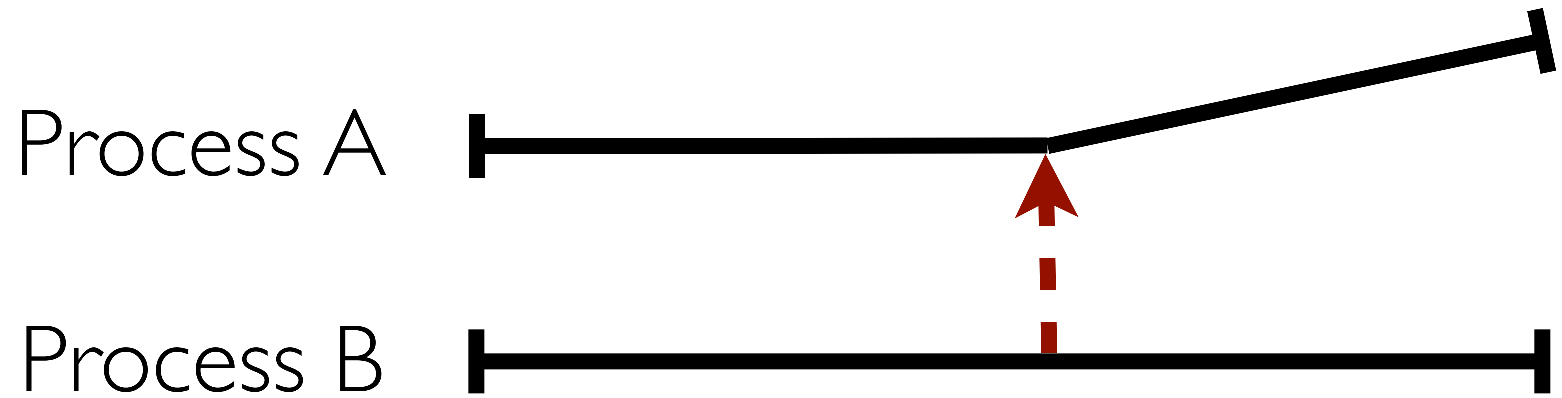
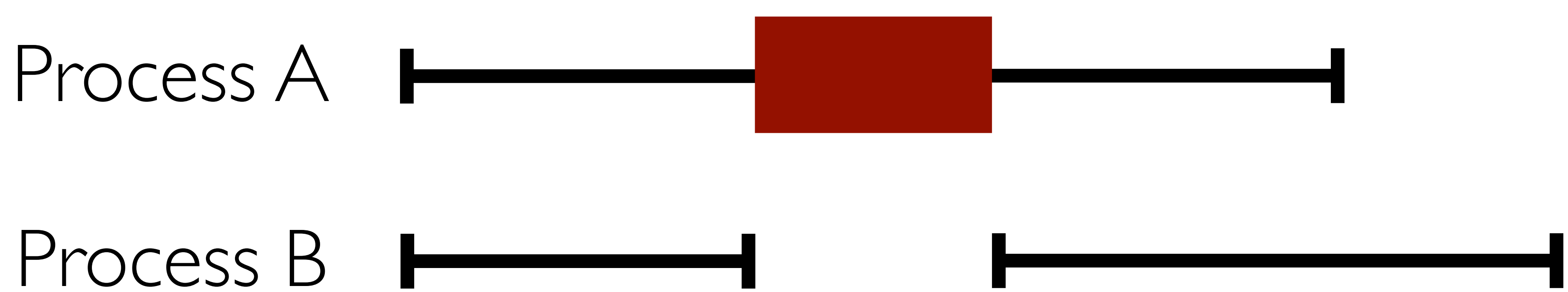
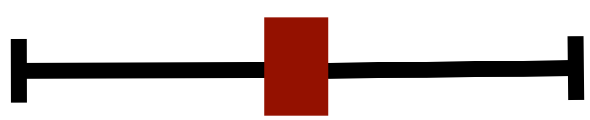


Atomicity is its antidote:



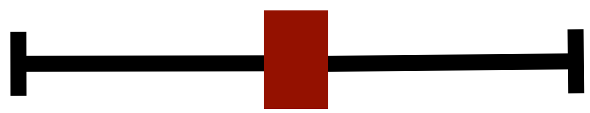
Granularity (in time & space)

Coarse-grained atomicity:
easier reasoning



Fine-grained atomicity:
lower latency
higher throughput (if parallel)

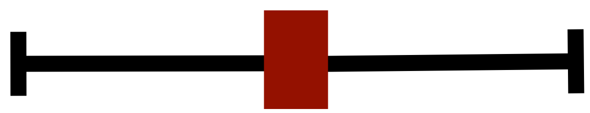
Atomicity abstraction



Fine-grained
implementation

Coarse-grained
specification

Atomicity abstraction



Fine-grained implementation

$\leq$

Coarse-grained specification

Refines, i.e., contextually approximates

Contributions

Direct, insightful refinement method
(no linearizability!)

Handles *higher-order, polymorphic* languages

Scales to *sophisticated* algorithms

Key Idea

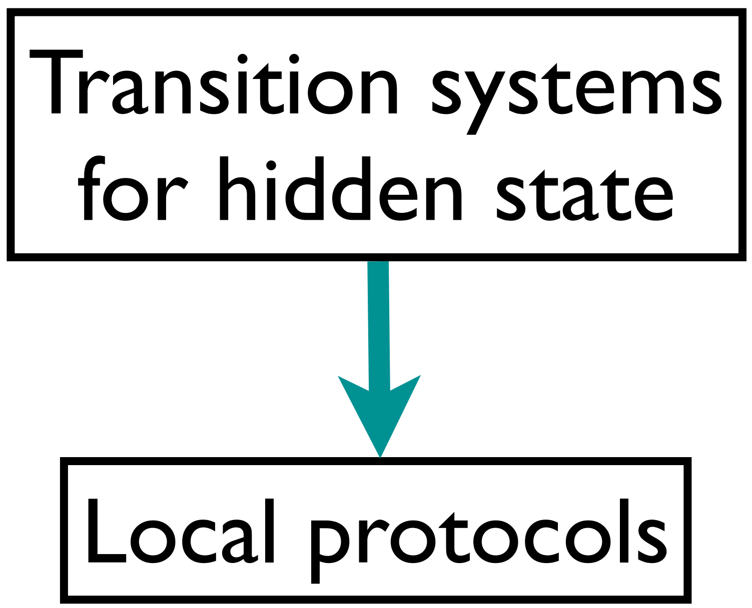
Atomicity abstraction, like data abstraction, relies on hiding

Approach

Transition systems for hidden state

State of the art: Kripke Logical Relations, ICFP10

Approach



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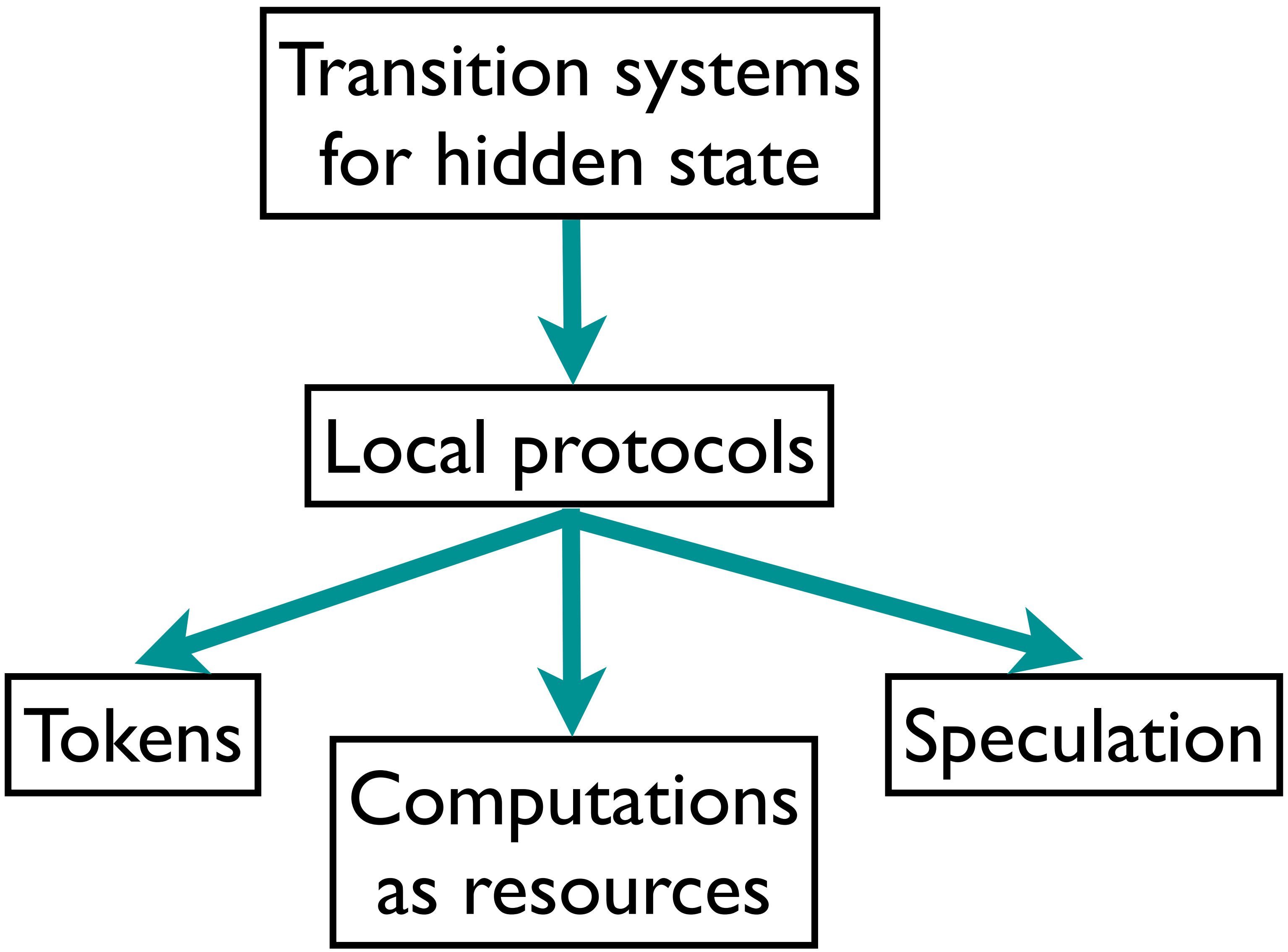
Fine granularity

Approach

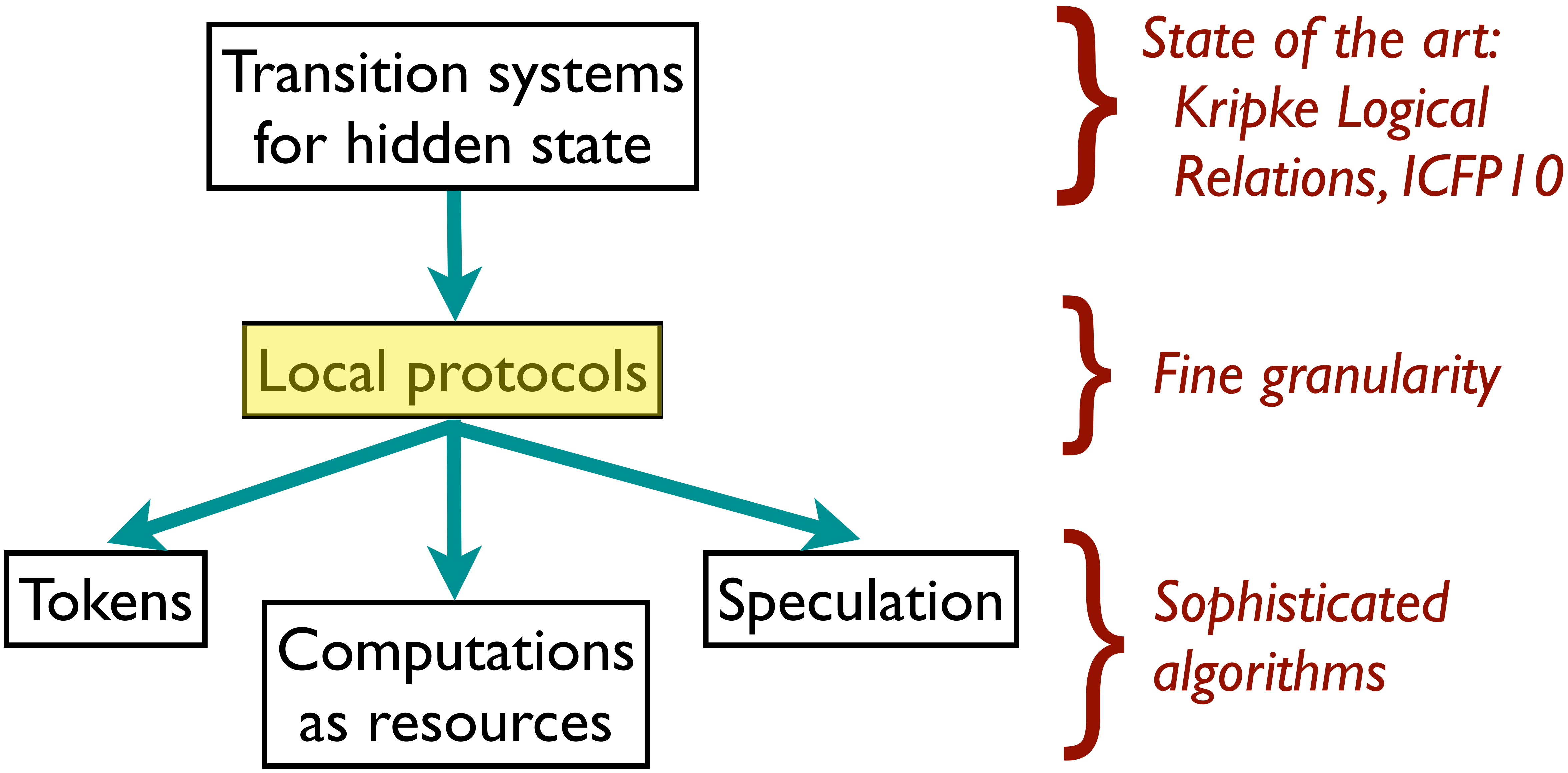


State of the art: Kripke Logical Relations, ICFP10

Fine granularity

Sophisticated algorithms

Approach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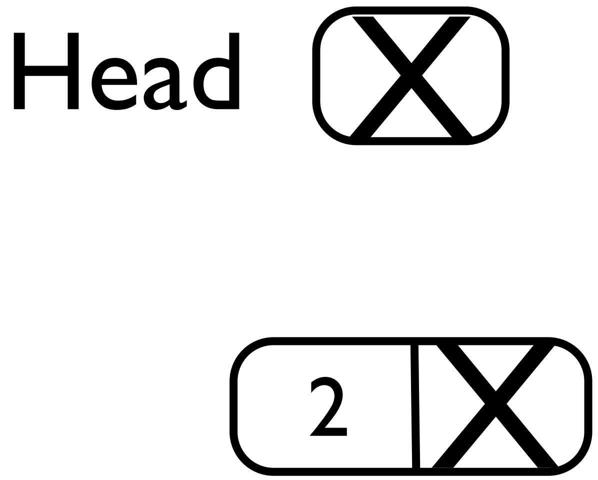
Compare and set

```
cas(cell, old, new) = atomic {
    if !cell := old
    then cell := new; true
    else false
}
```

A Lock-free "Queue"

A Lock-free “Queue”



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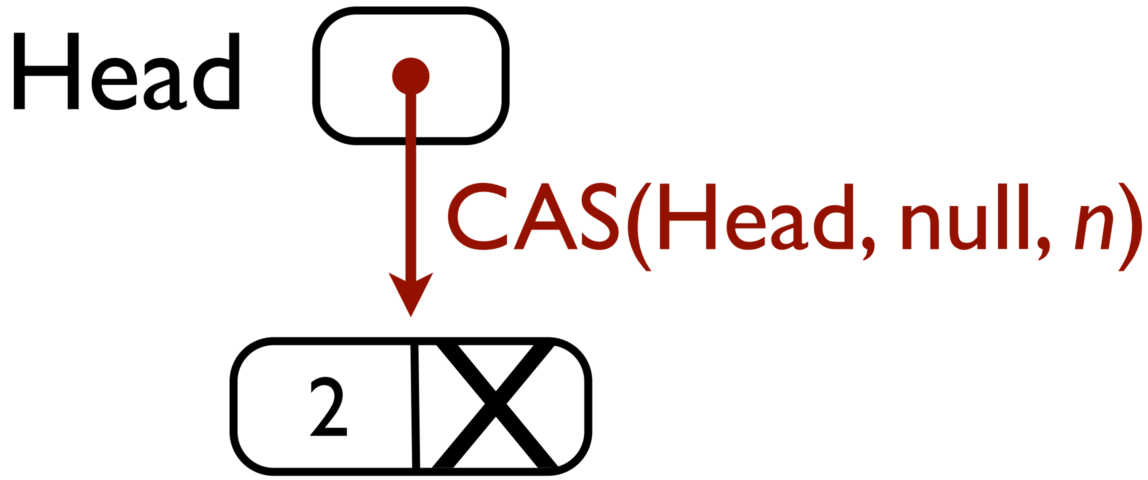

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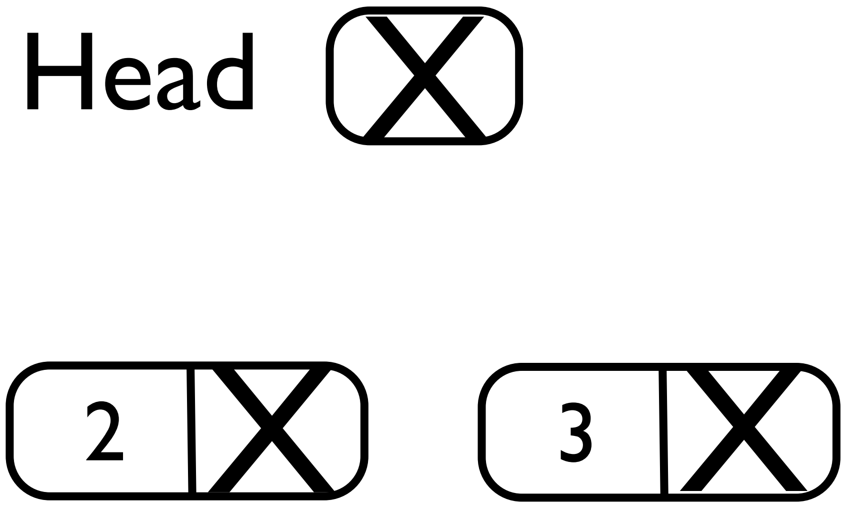

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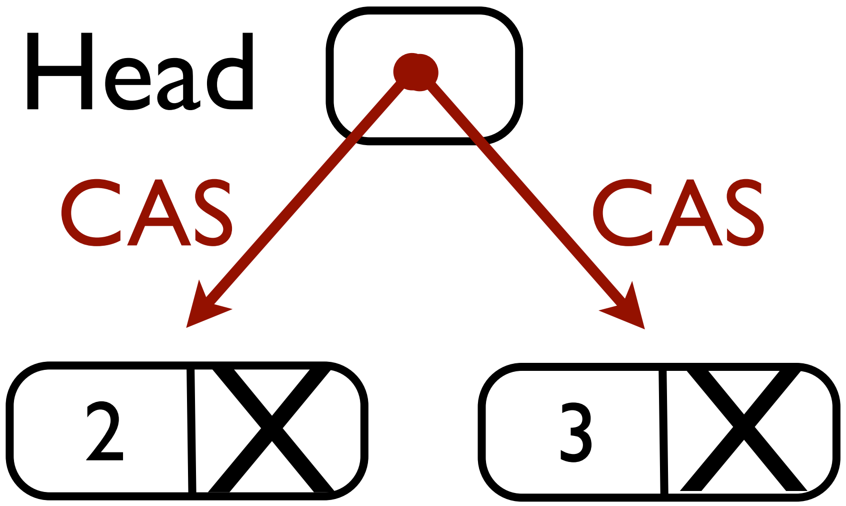

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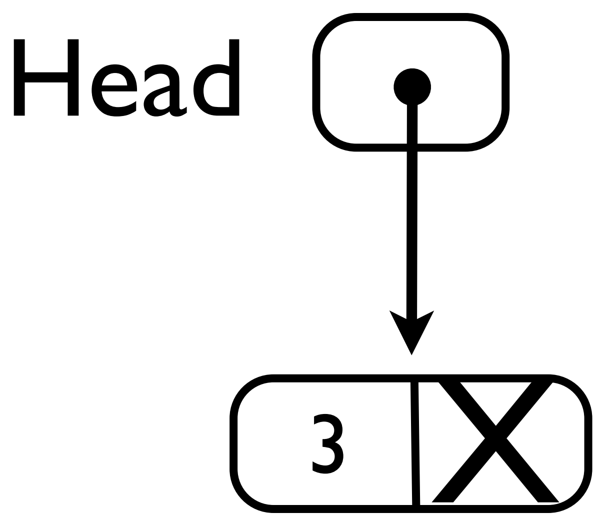

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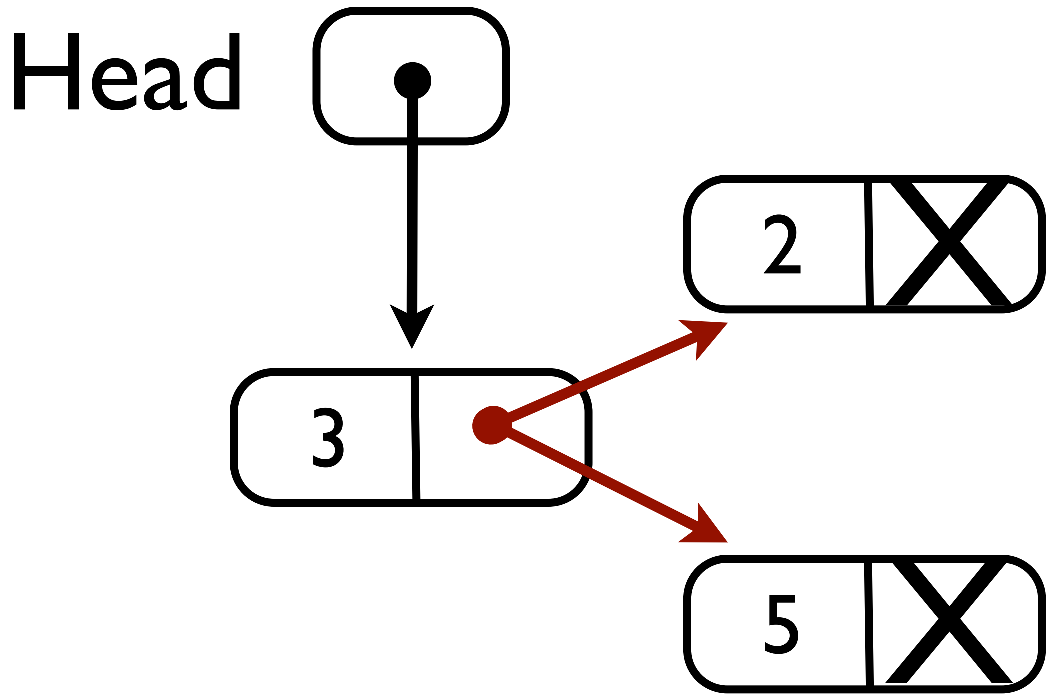

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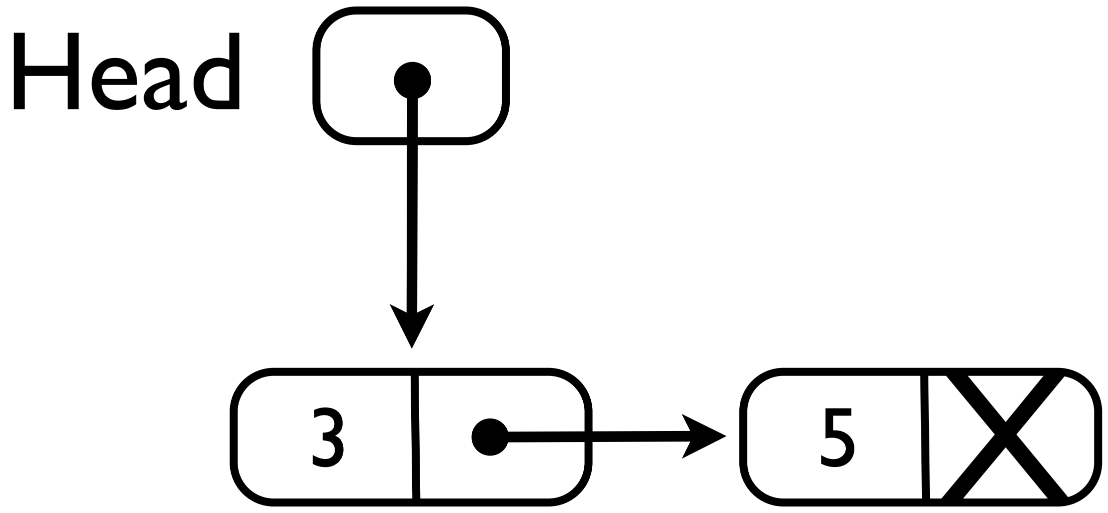

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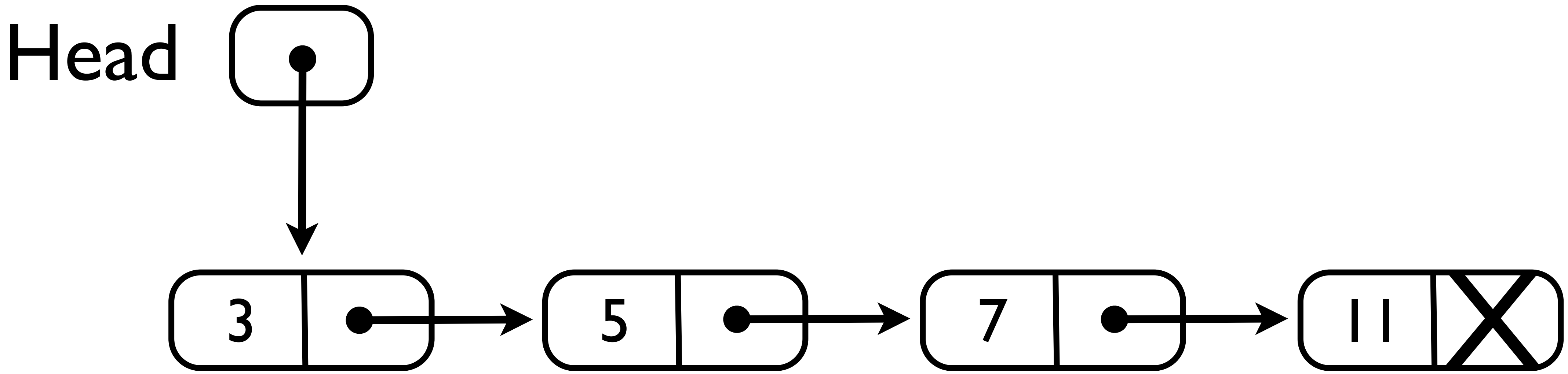

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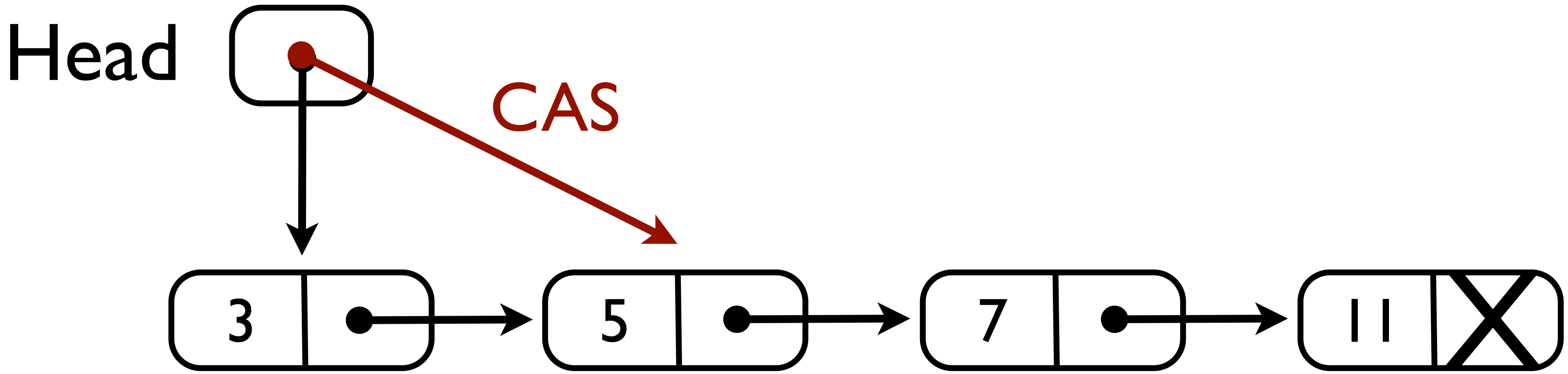

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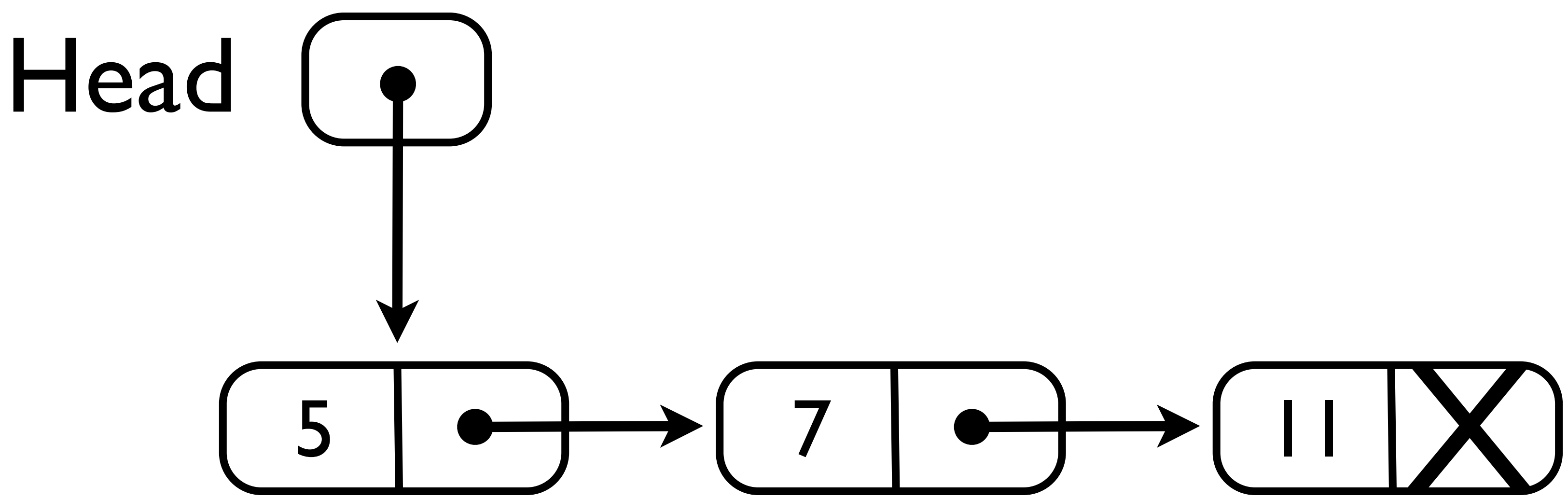

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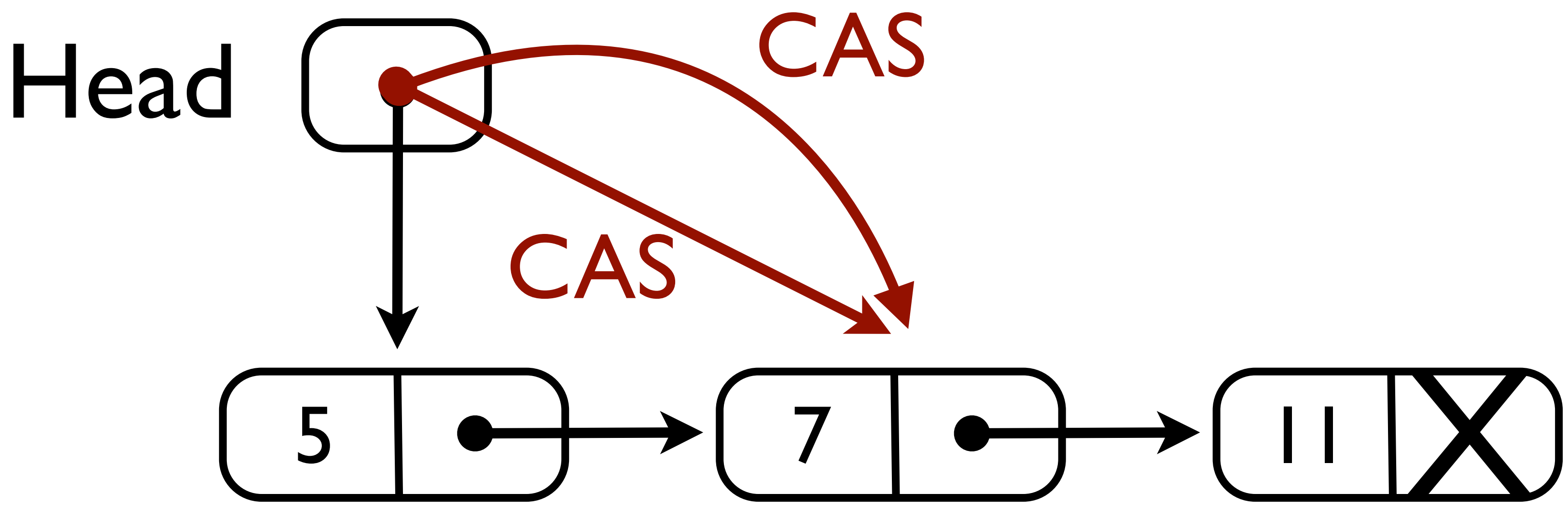

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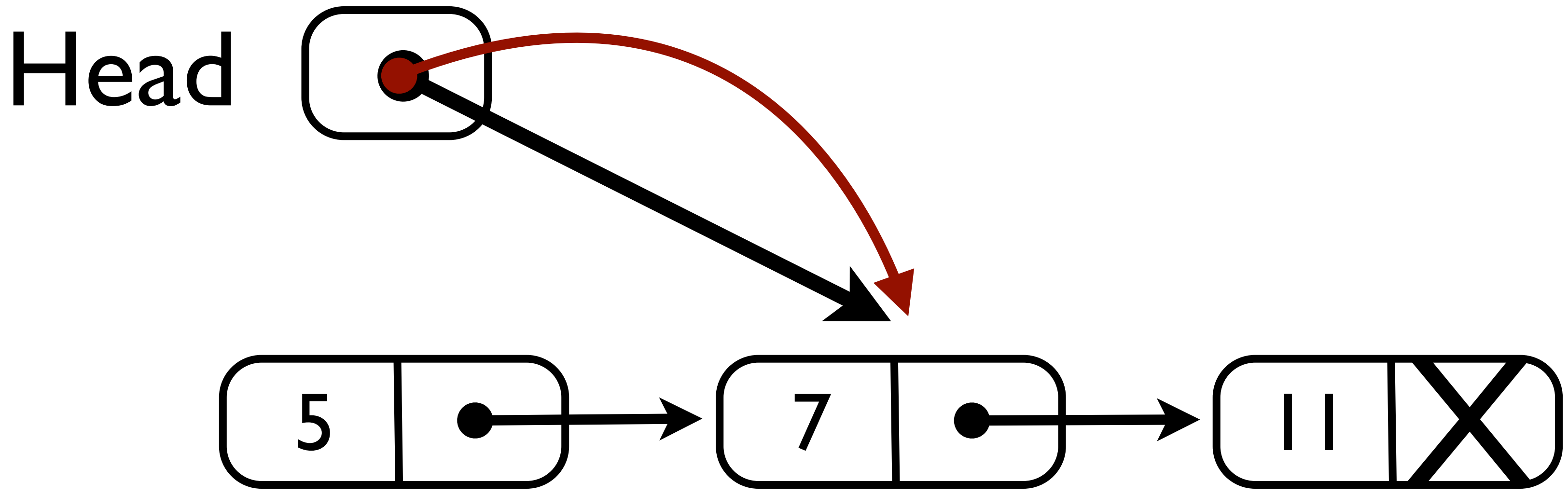

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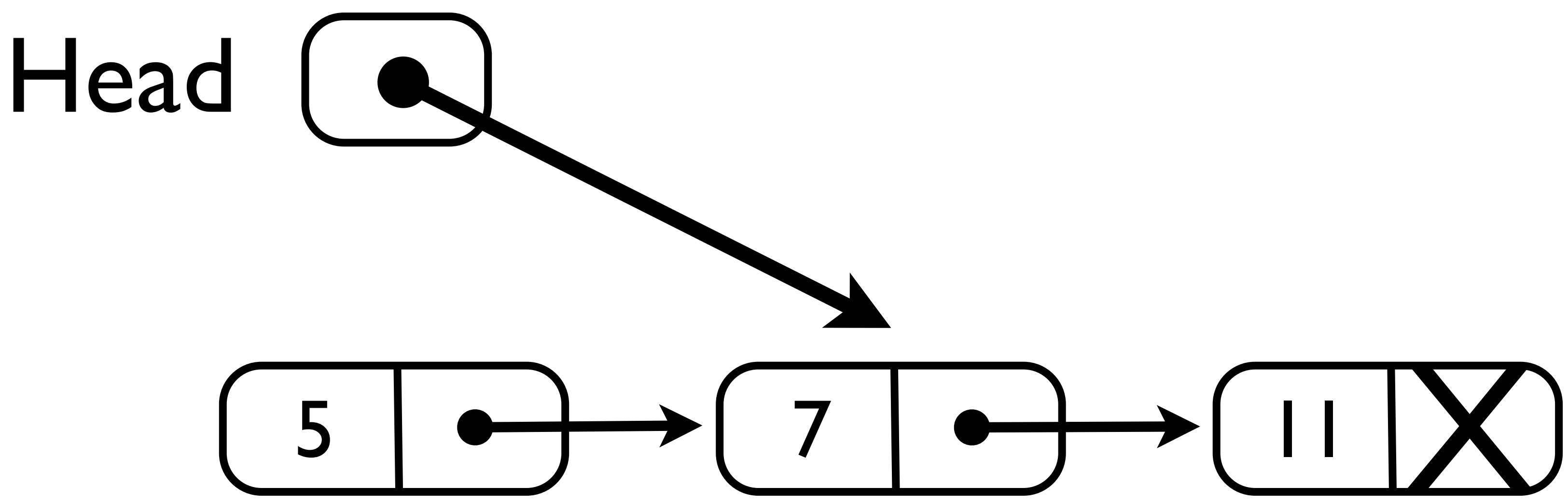

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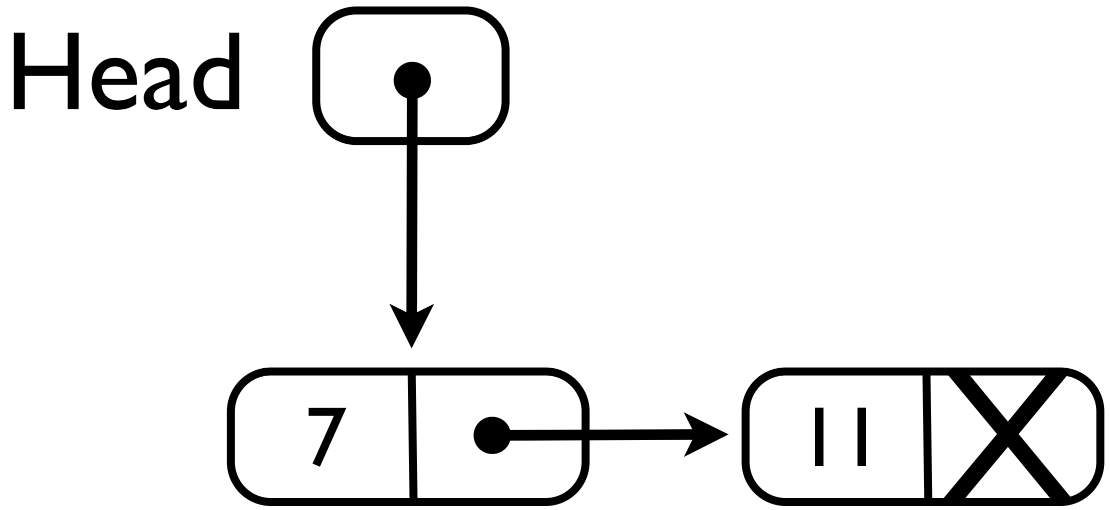

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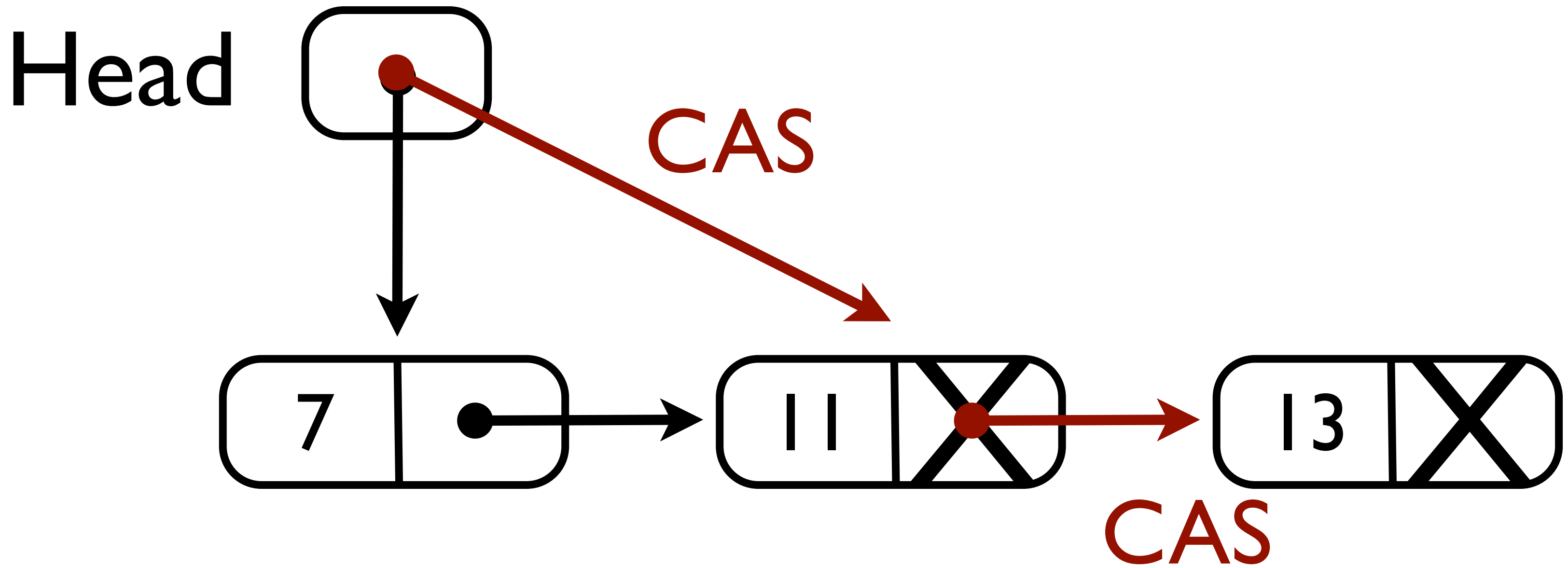

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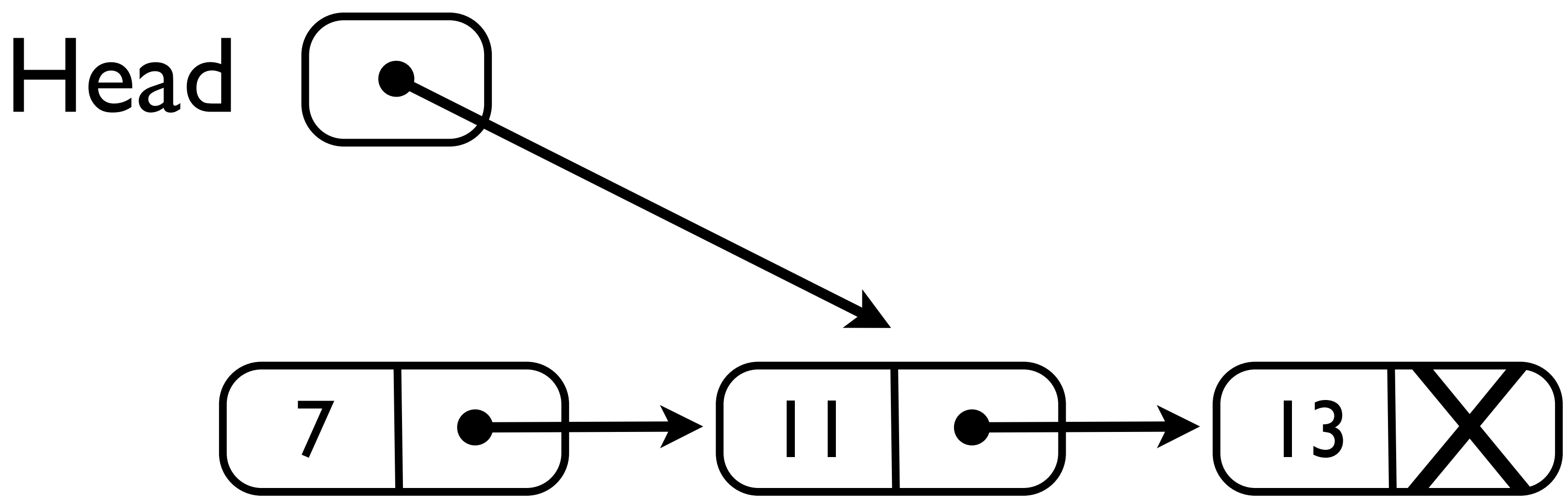

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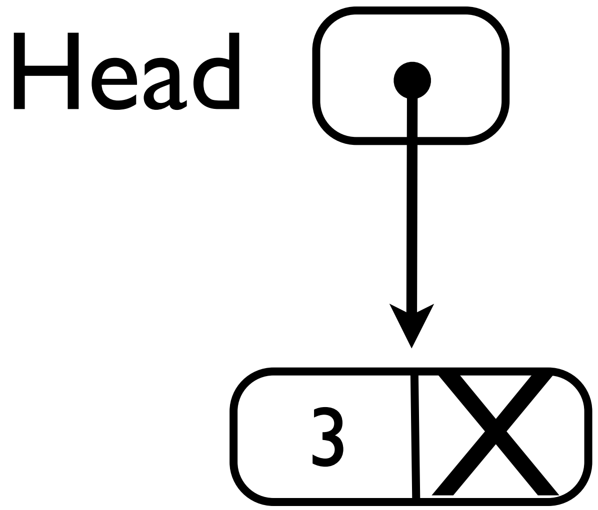

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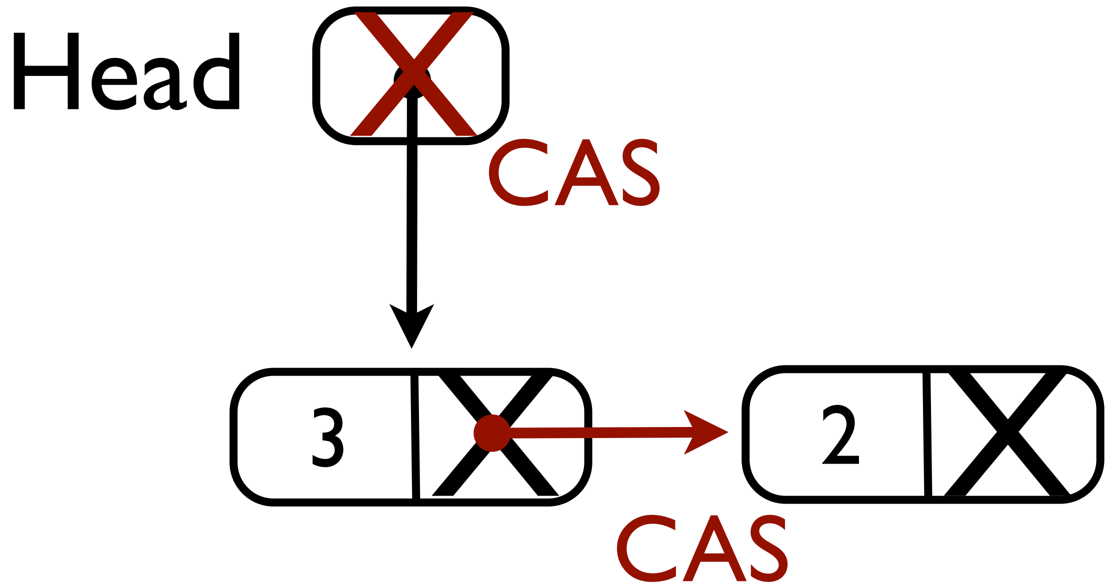

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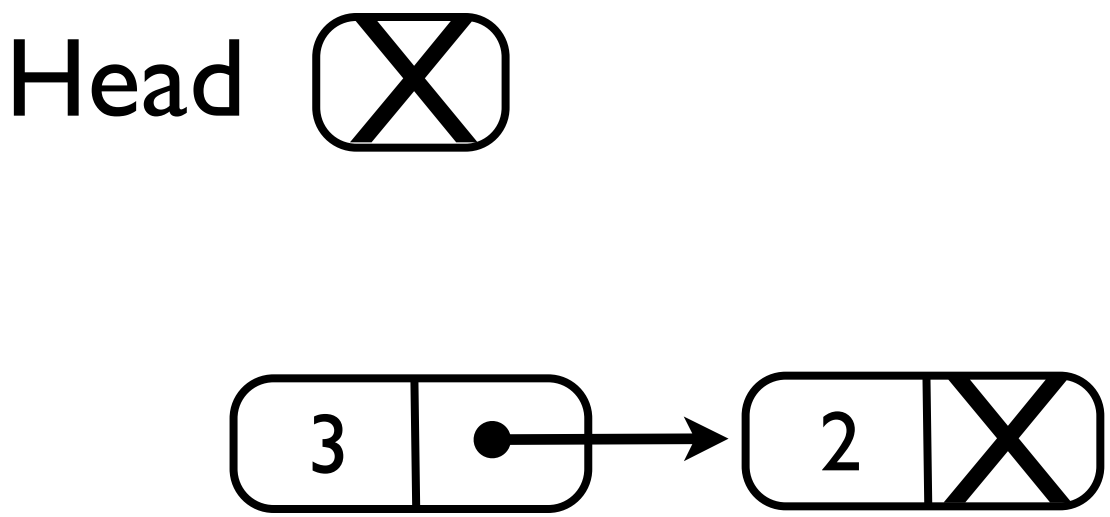

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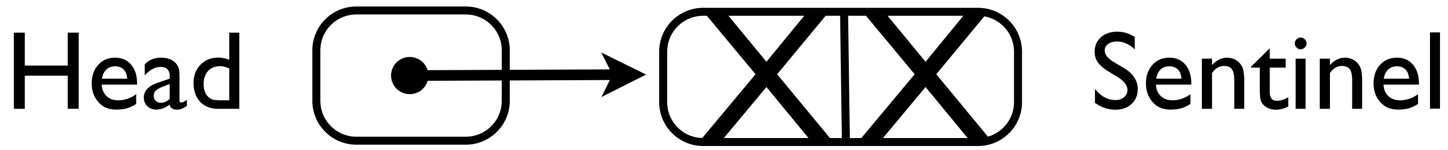

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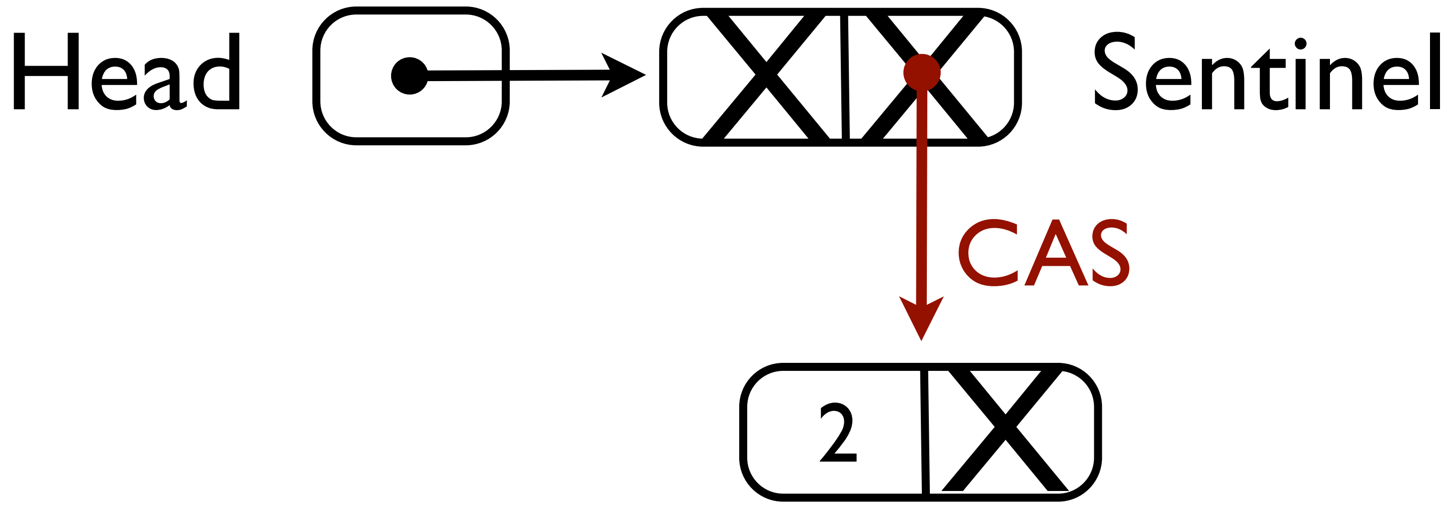

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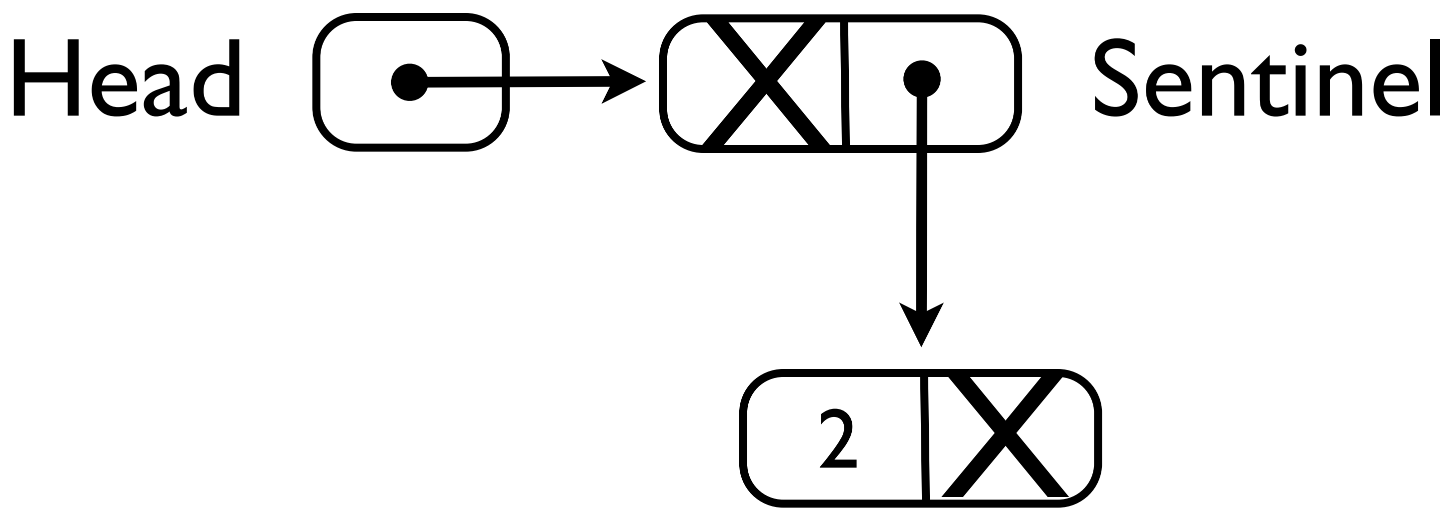

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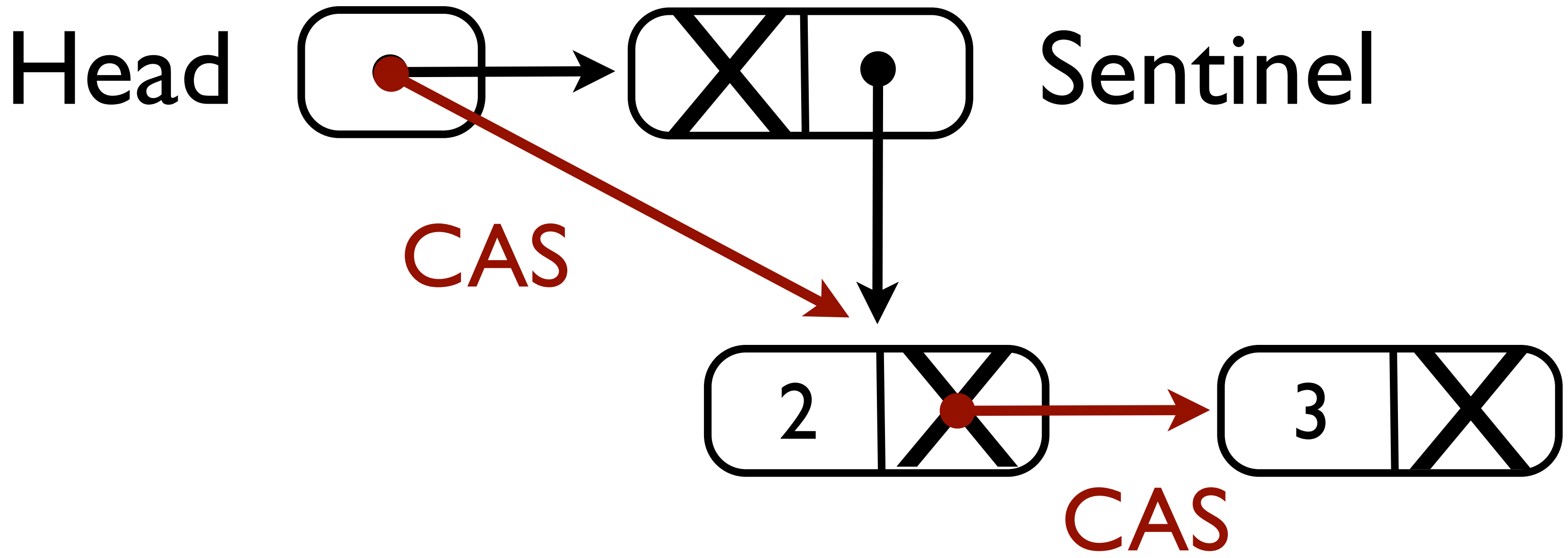

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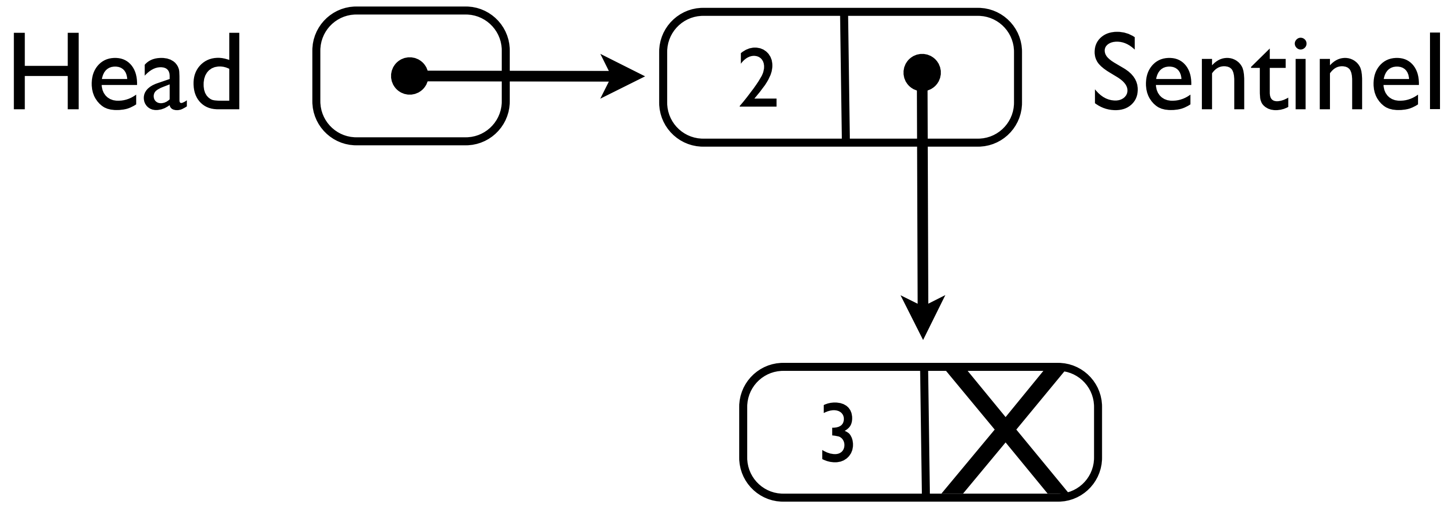
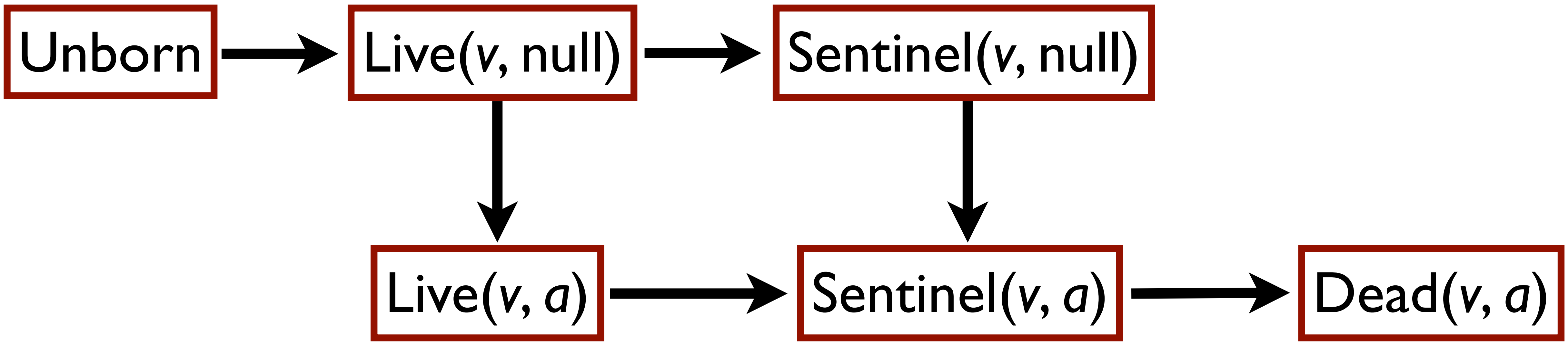


The life story of a node



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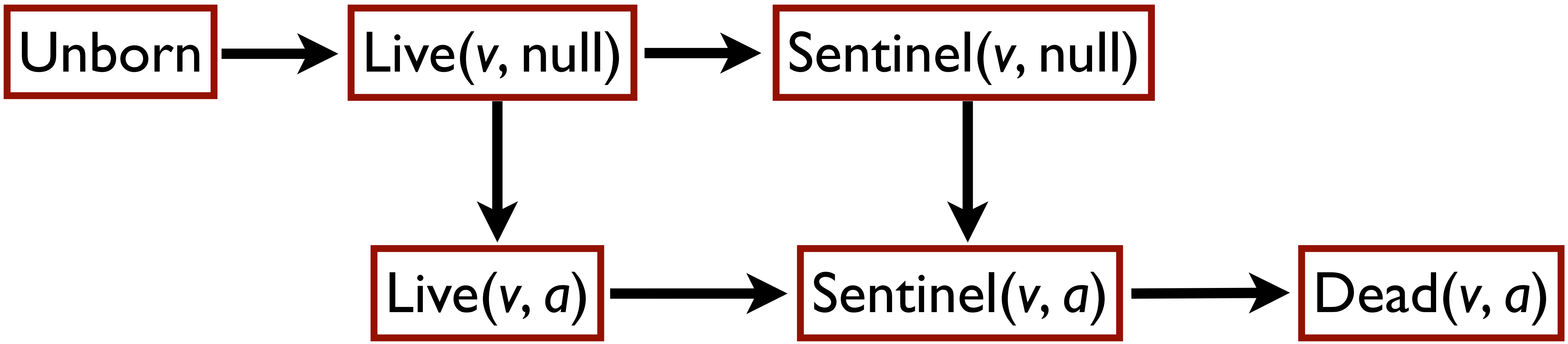

Global constraints:

Exactly one sentinel

Live iff reachable from sentinel

Nonnull next pointers are born

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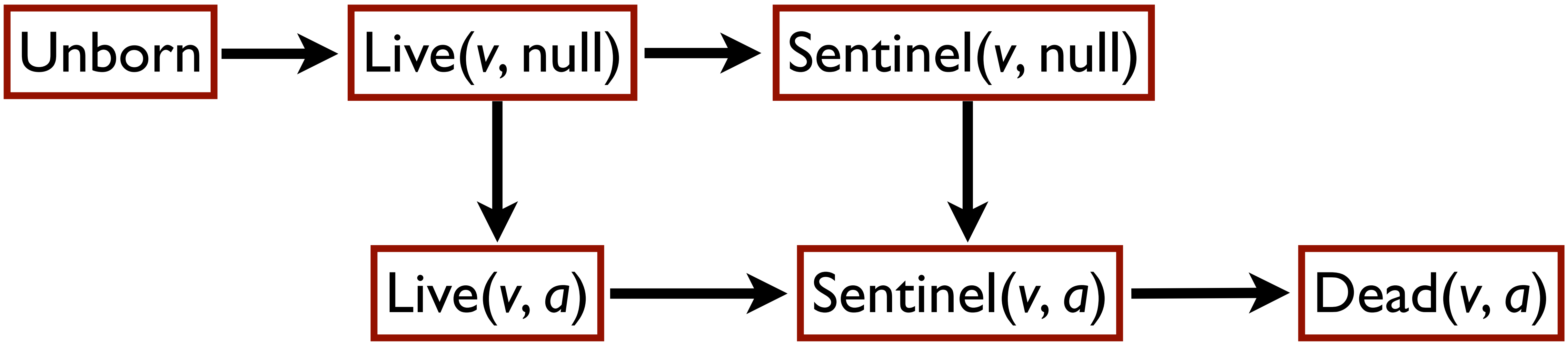

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linkNode(p, n) = case !(p.next)
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        if cas(p.next, null, n)
        then () else linkNode(p, n)
  | p’ ⇒ linkNode(p’, n)
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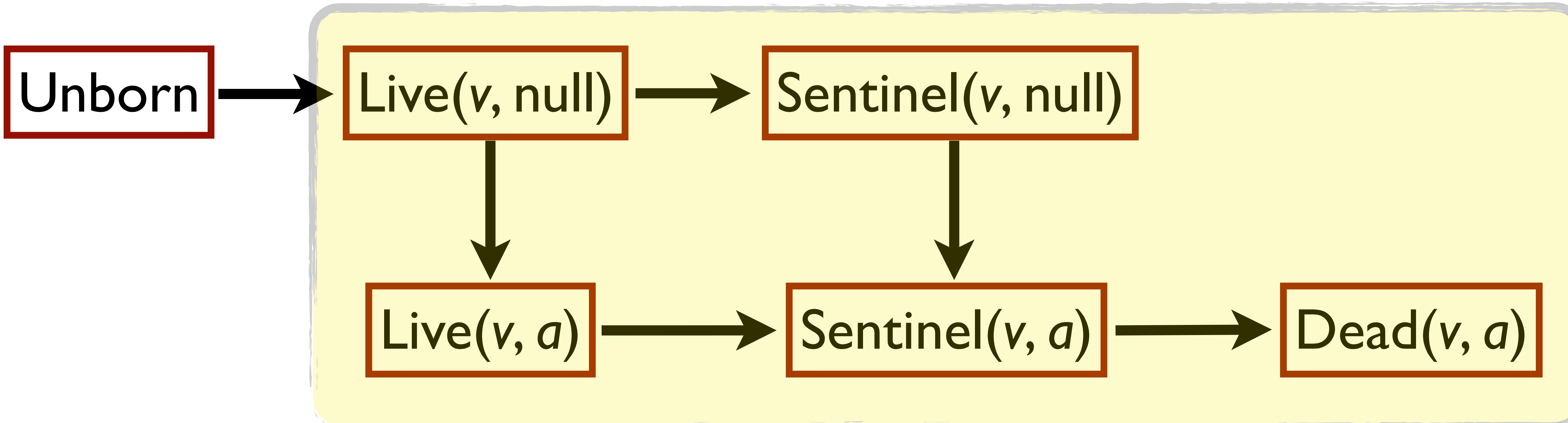

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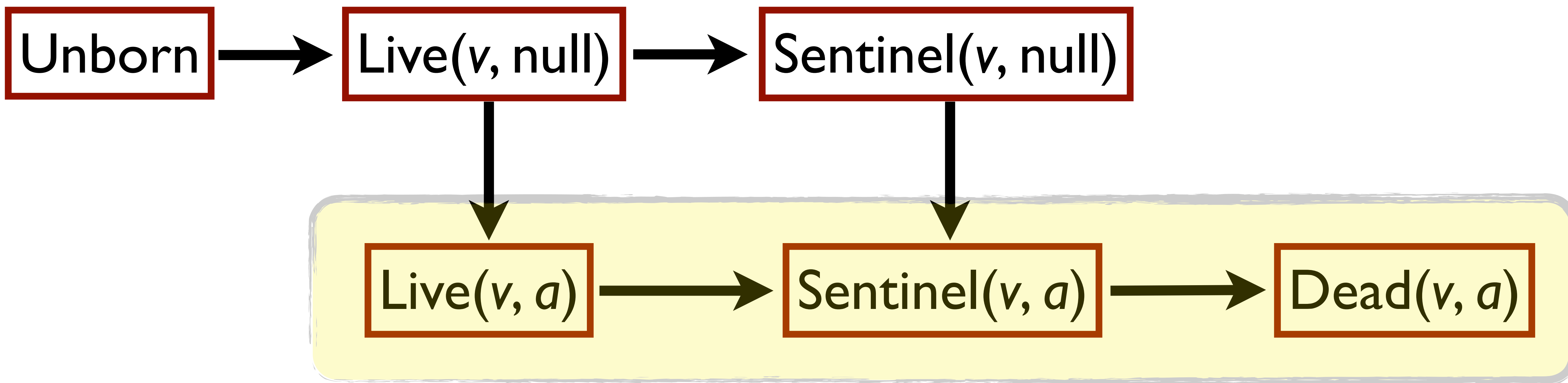

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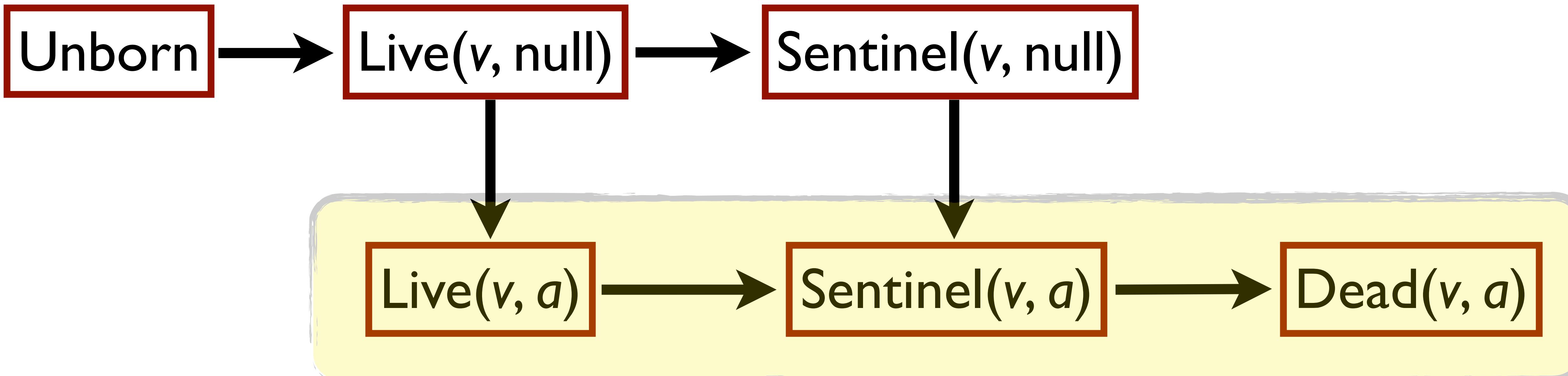

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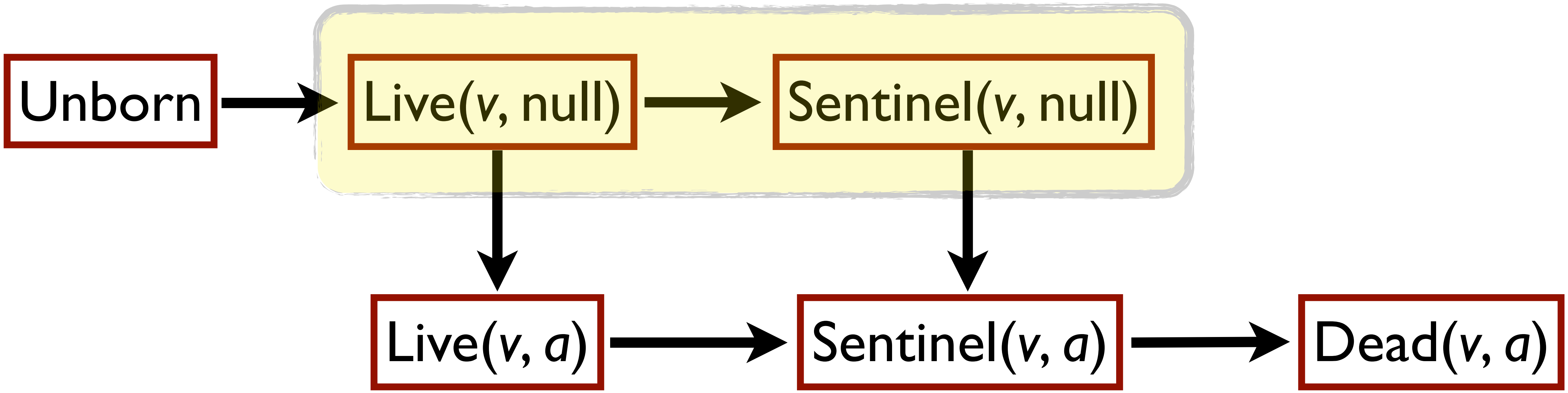

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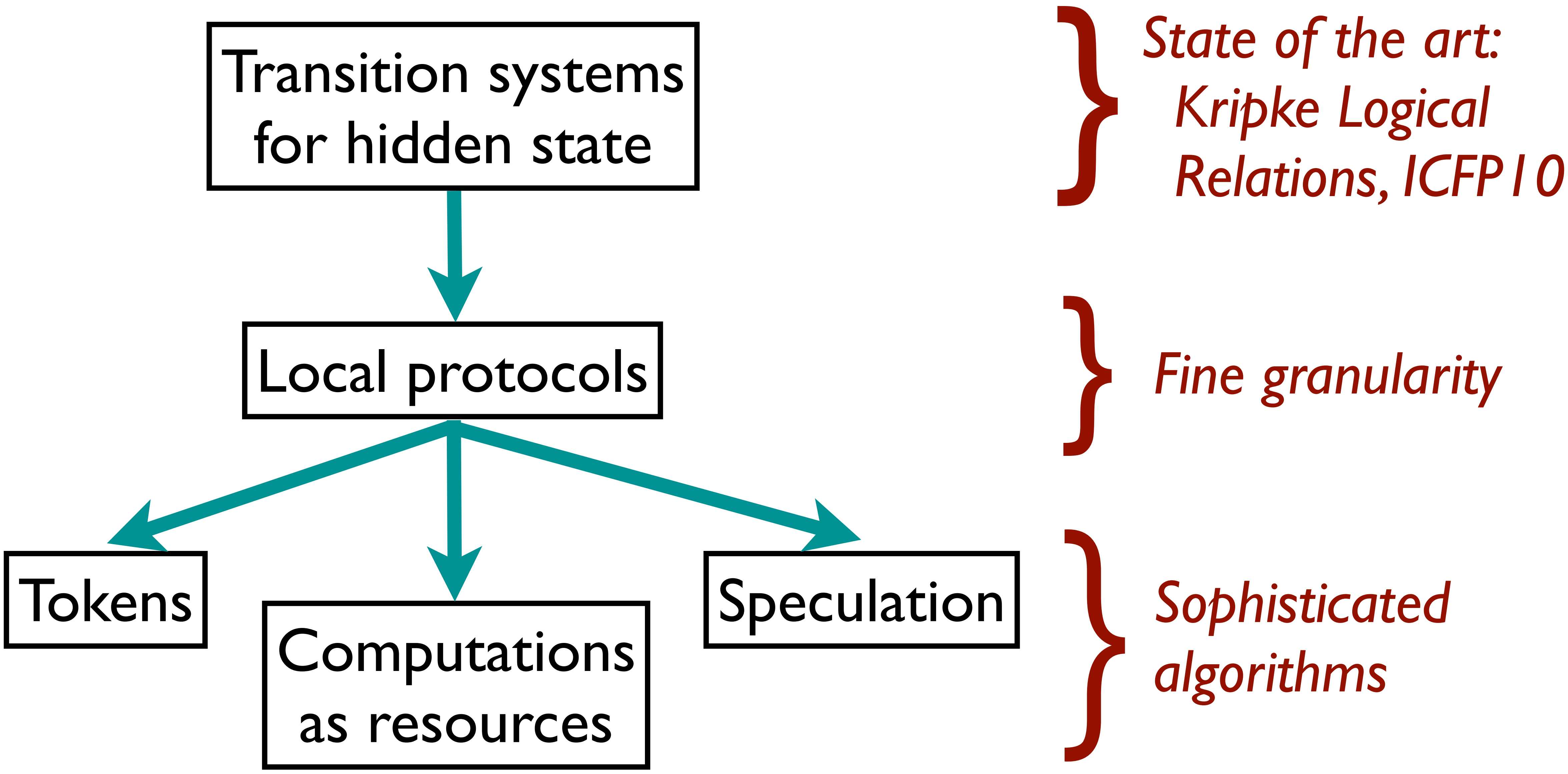
```

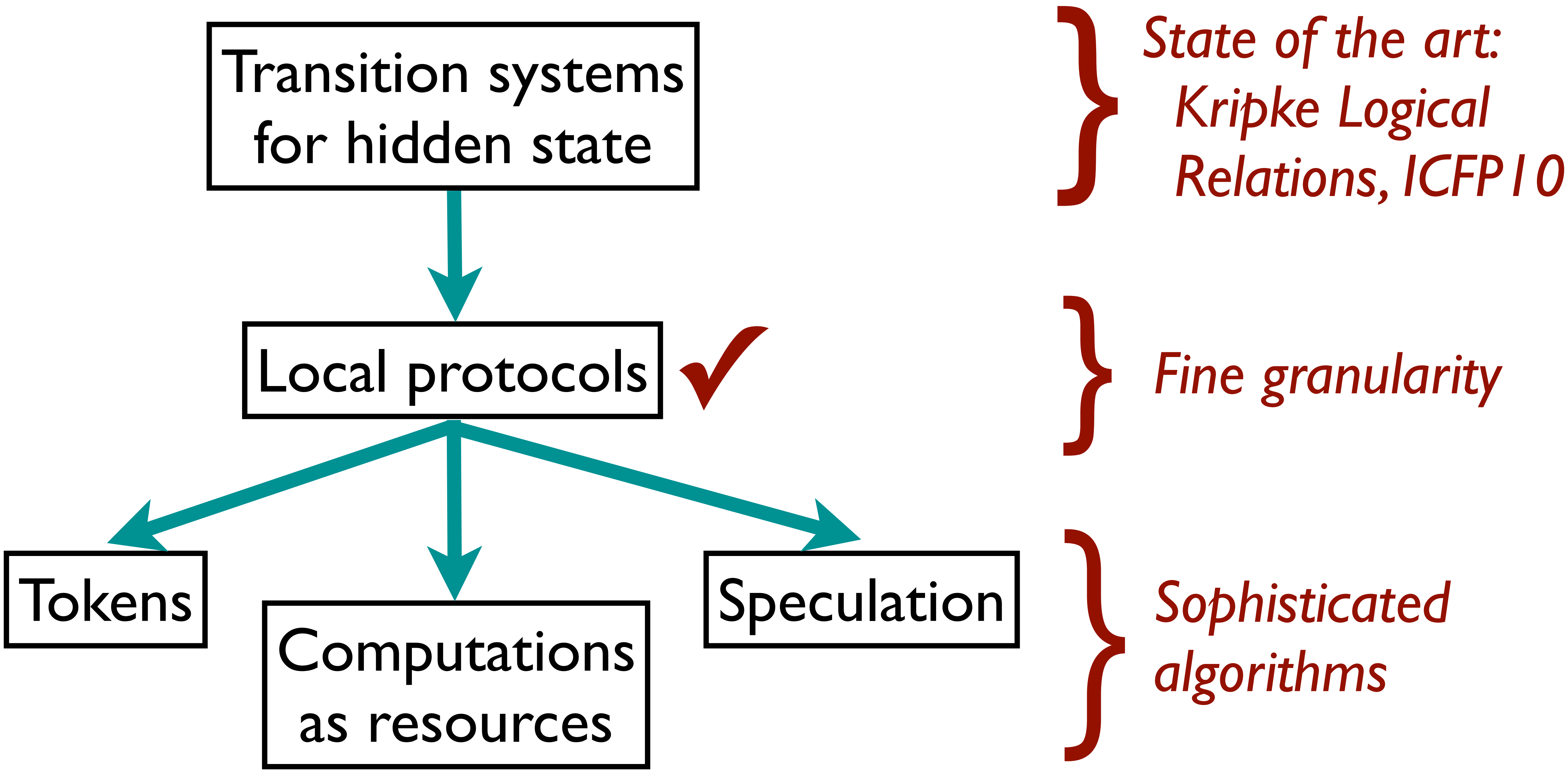
Key Idea

Local protocols capture abstract knowledge/interference at the same granularity as the algorithm operates

Approach


Transition systems for hidden state
Local protocols
Tokens
Computations as resources
Speculation
State of the art: Kripke Logical Relations, ICFP10
Fine granularity
Sophisticated algorithms

Approach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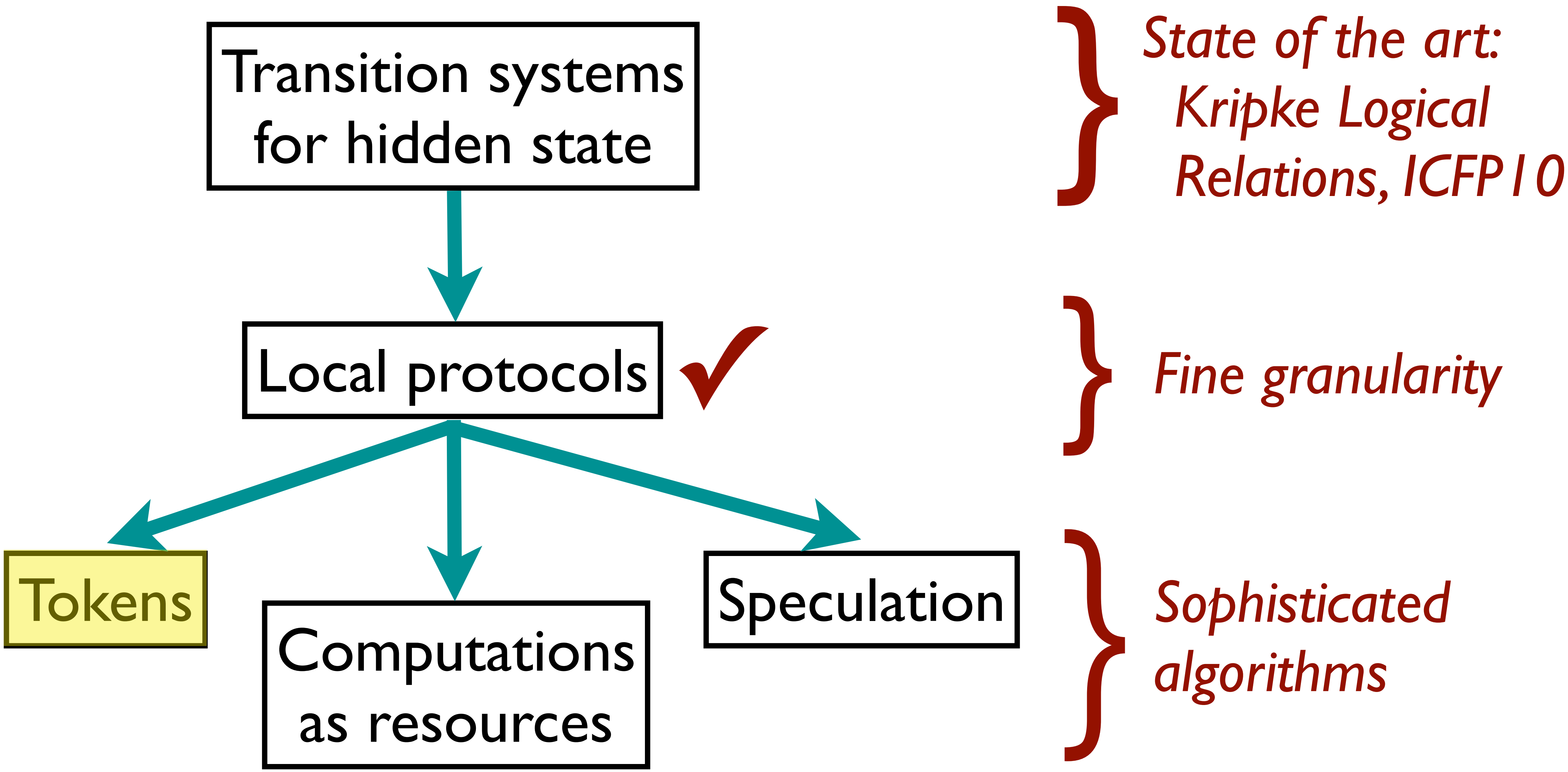
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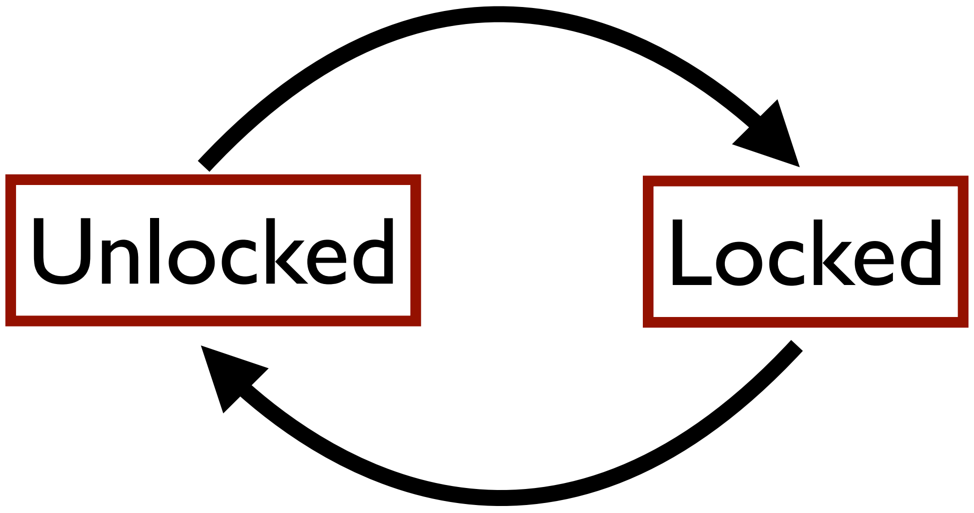
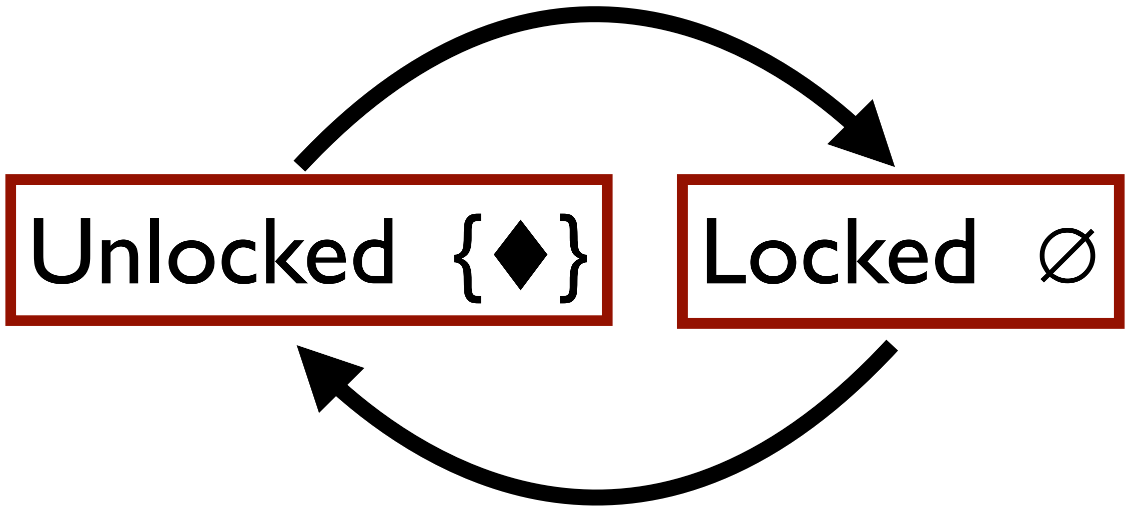

Unlocked
Locked


Unlocked {♦}
Locked ∅

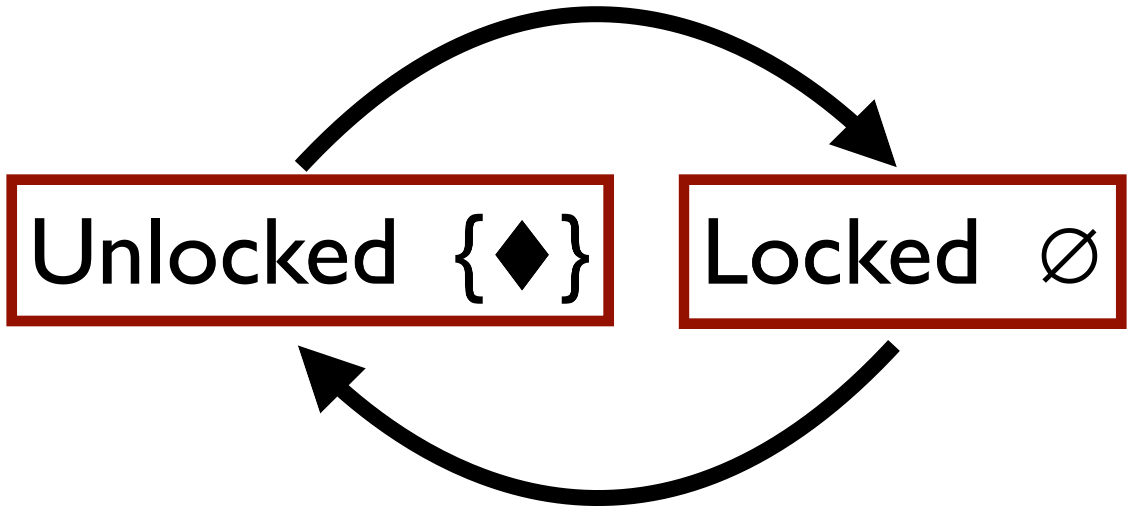
Conservation Law

Before *After*

$$T \uplus P = T' \uplus P'$$

Thread's tokens

Protocol's tokens



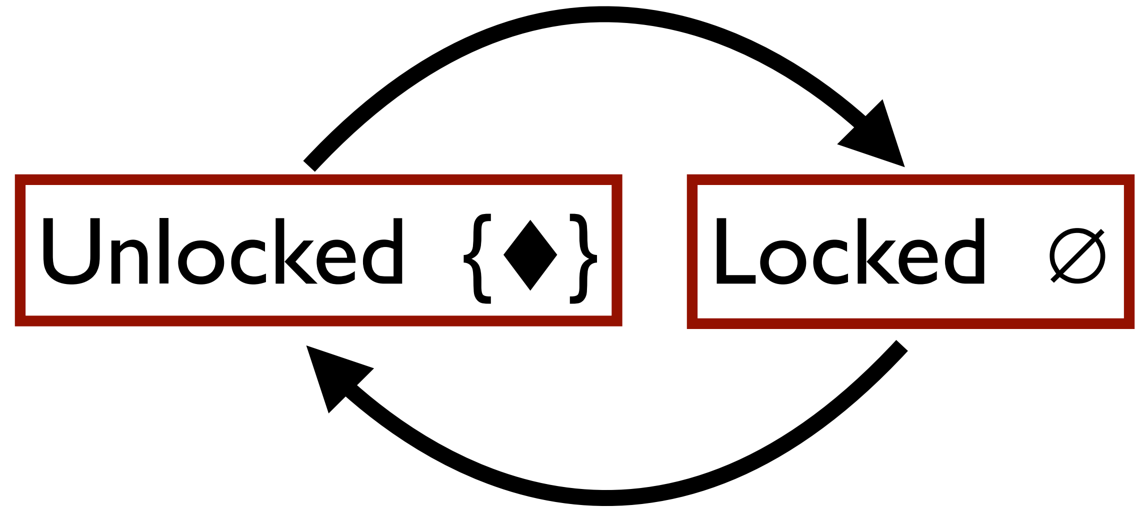
Conservation Law

Before *After*

$T \uplus P = T' \uplus P'$

Thread's tokens (→ T)

Protocol's tokens (→ P)



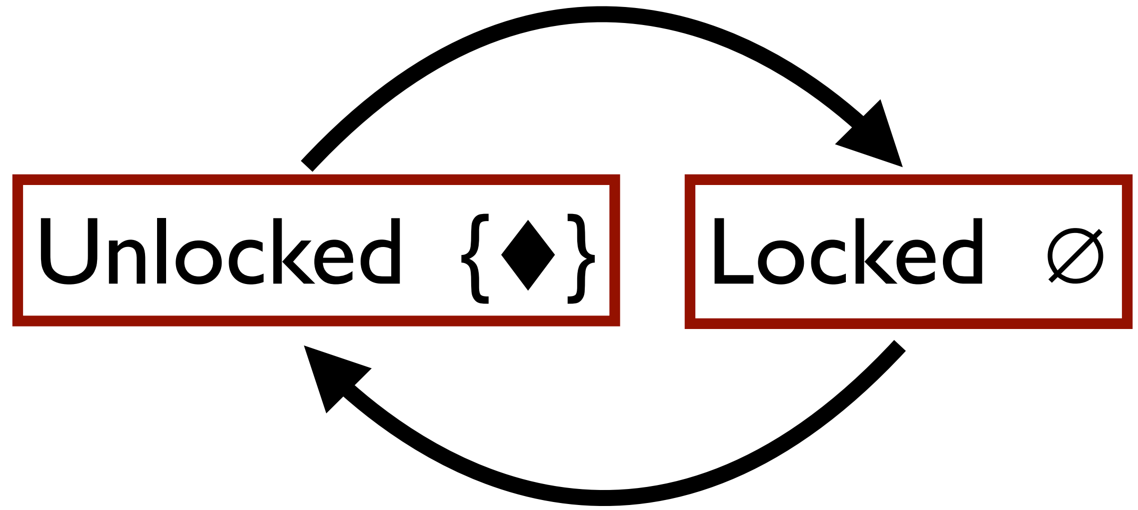
Unlocked → Locked: $\underbrace{\emptyset}_{T} \uplus \underbrace{\{\blacklozenge\}}_{P} = \underbrace{?}_{T'} \uplus \underbrace{\emptyset}_{P'}$

Conservation Law

Before *After*

$$T \uplus P = T' \uplus P'$$

Thread's tokens (T) — Protocol's tokens (P)



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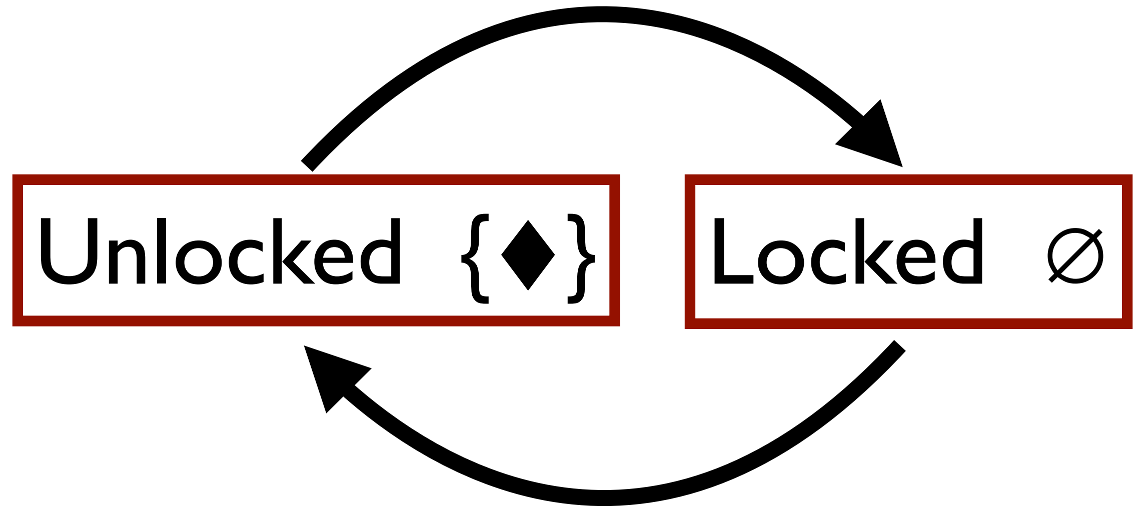


T P T' P'

Unlocked → Locked: $\emptyset \uplus \{\blacklozenge\} = \{\blacklozenge\} \uplus \emptyset$

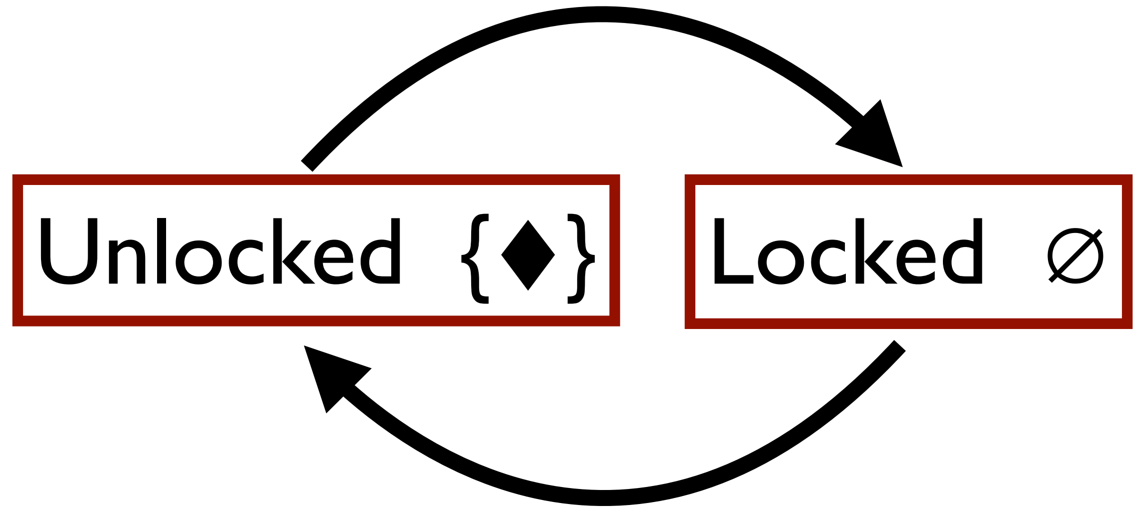
Conservation Law

Before *After*

$$T \uplus P = T' \uplus P'$$

T: Thread's tokens

P: Protocol's tokens



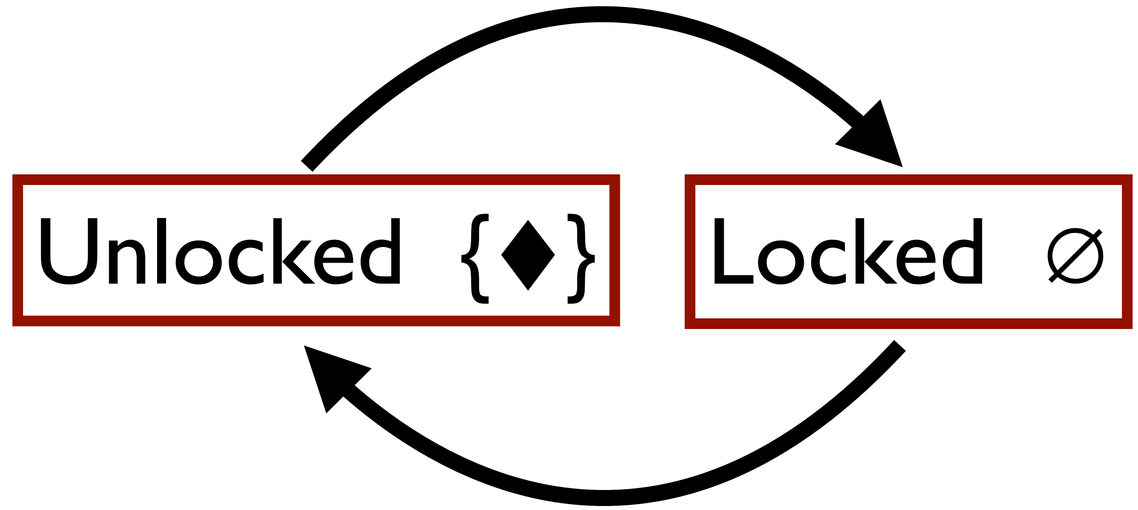
	T		P		T'		P'
Unlocked → Locked:	$\emptyset$	$\uplus$	$\{\blacklozenge\}$	$=$	$\{\blacklozenge\}$	$\uplus$	$\emptyset$
Locked → Unlocked:	$\emptyset$	$\uplus$	$\emptyset$	$=$	**?**	$\uplus$	$\{\blacklozenge\}$

Conservation Law

Before *After*

$$T \uplus P = T' \uplus P'$$

Thread's tokens ↗ (T)   Protocol's tokens ↖ (P)



T P T' P'

Unlocked → Locked: $\emptyset \uplus \{\blacklozenge\} = \{\blacklozenge\} \uplus \emptyset$

~~Locked → Unlocked: $\emptyset \uplus \emptyset = ? \uplus \{\blacklozenge\}$~~ *Impossible*

Key Idea

Tokens enable threads to gain and lose roles *dynamically*, via transitions

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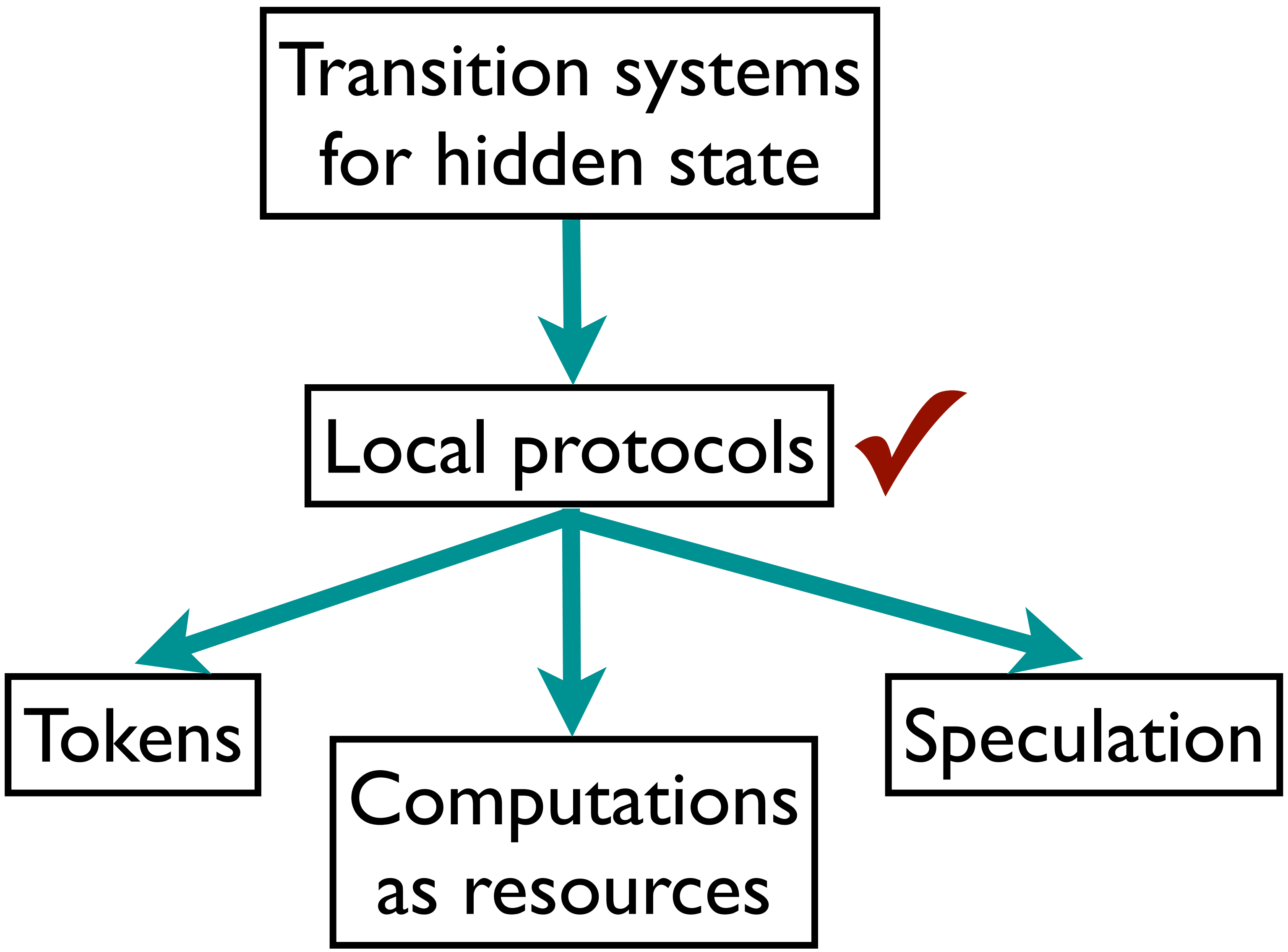
Tokens enable threads to gain and lose roles *dynamically*, via transitions

Rely: transitions with environment tokens
Guarantee: transitions with thread's tokens

Approach



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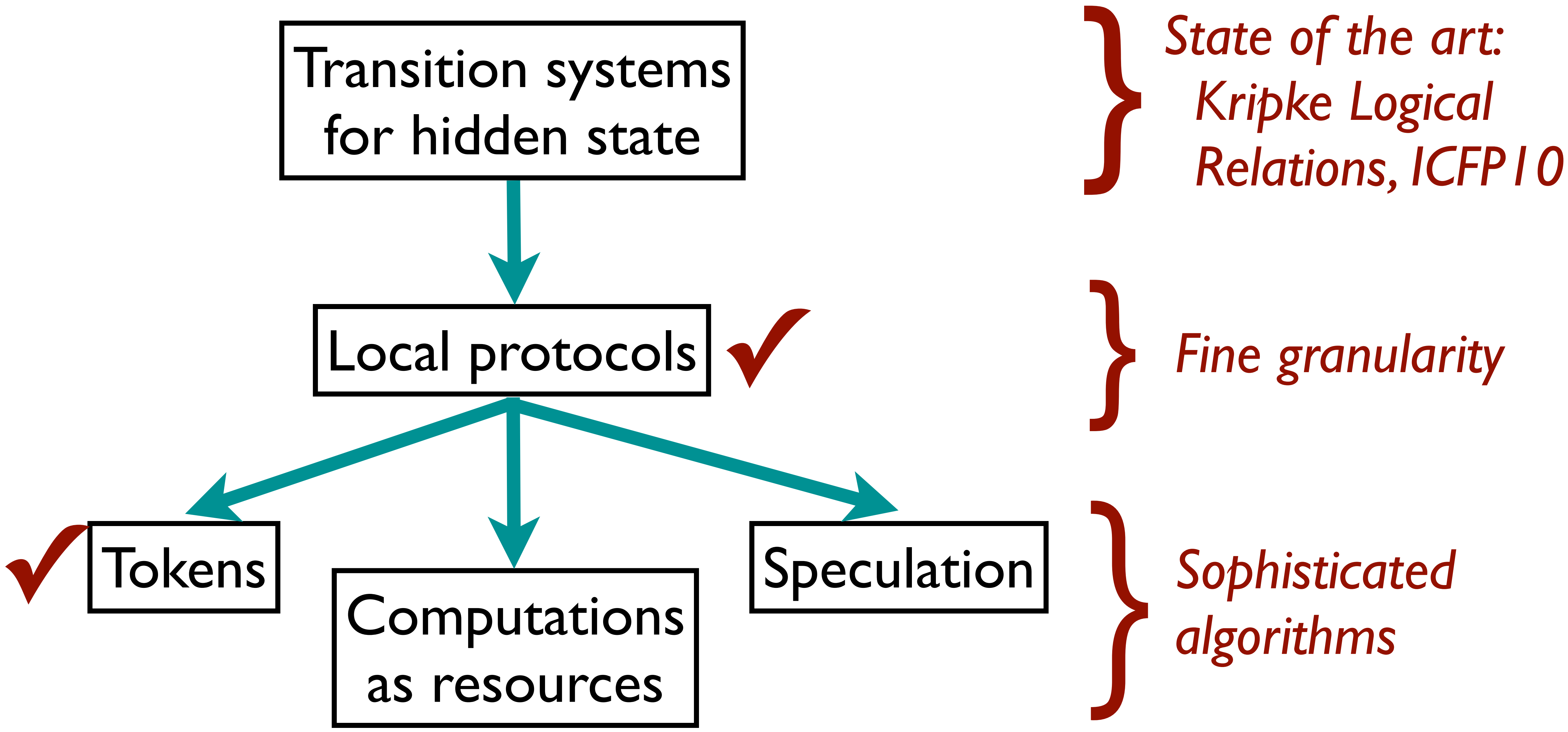
Fine granularity

Sophisticated algorithms

Approach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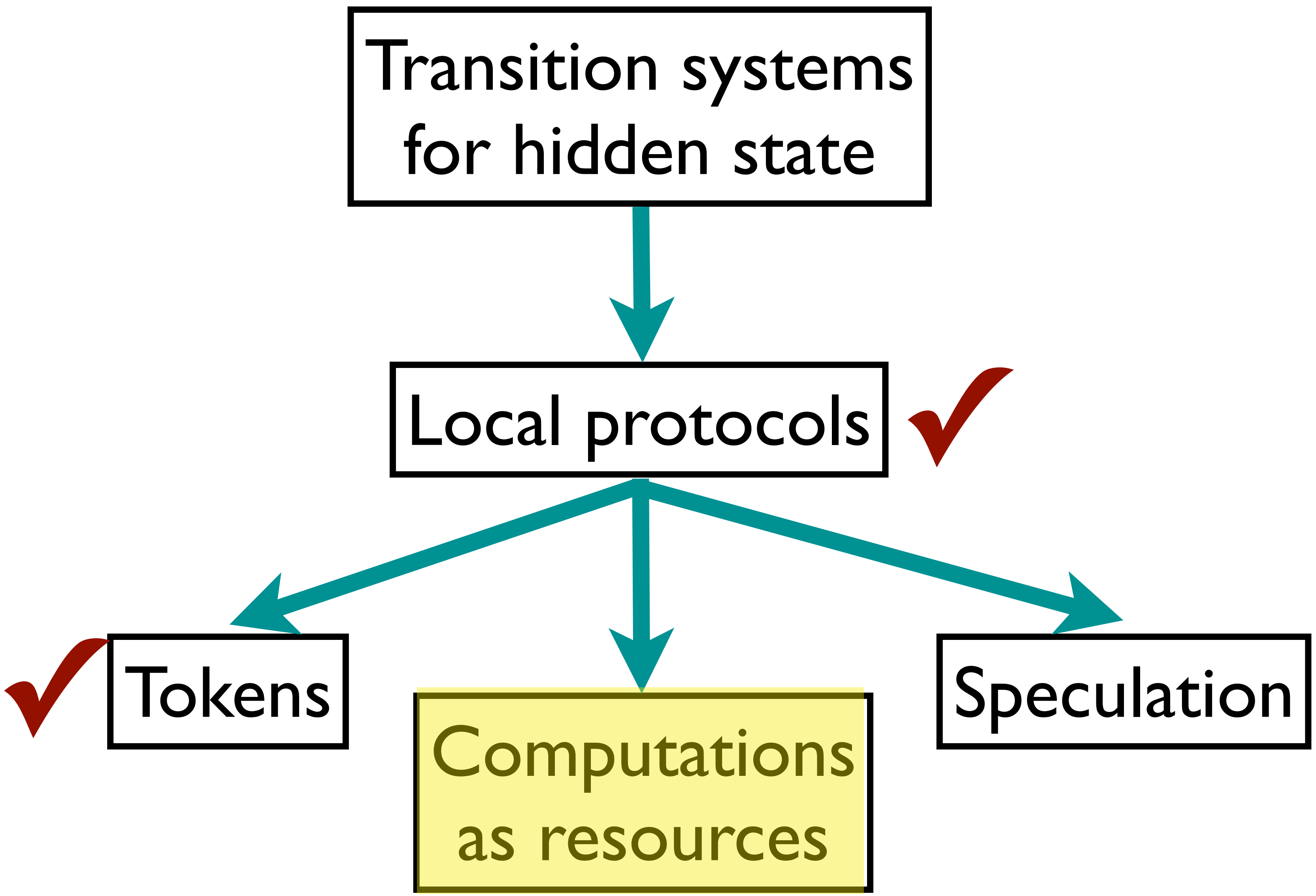

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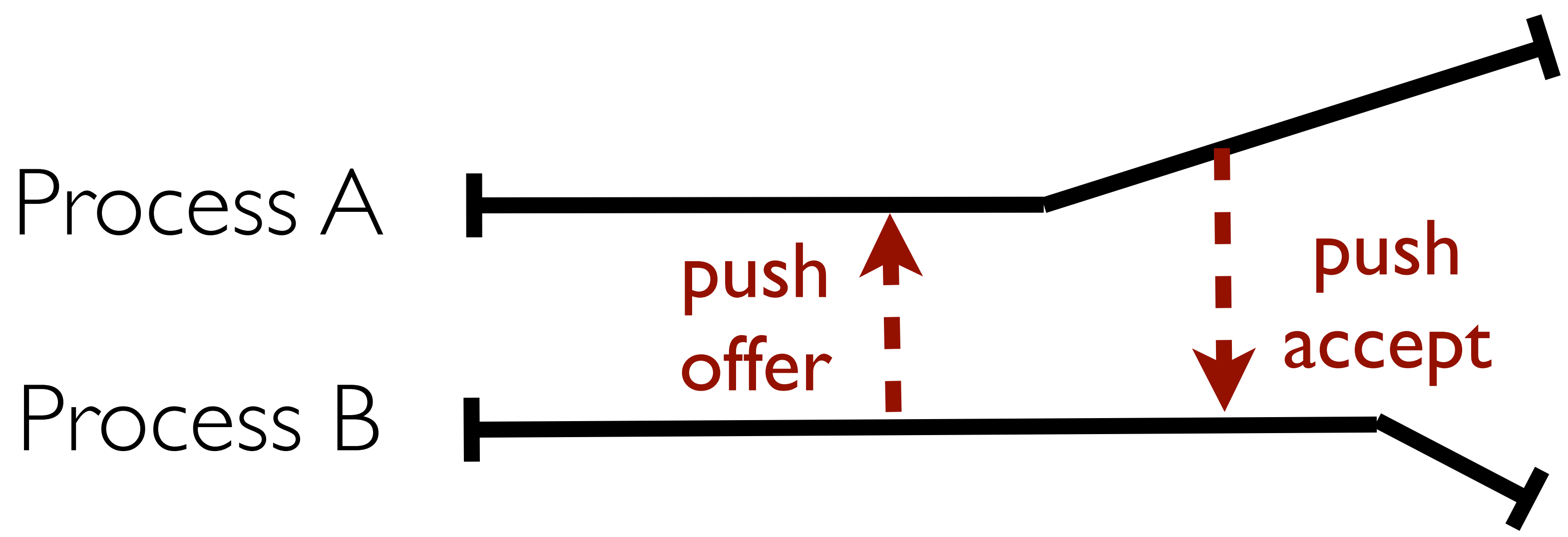
Fine granularity

Sophisticated algorithms

Problem:

Cooperation is inherently non-thread-local

e.g. elimination stacks



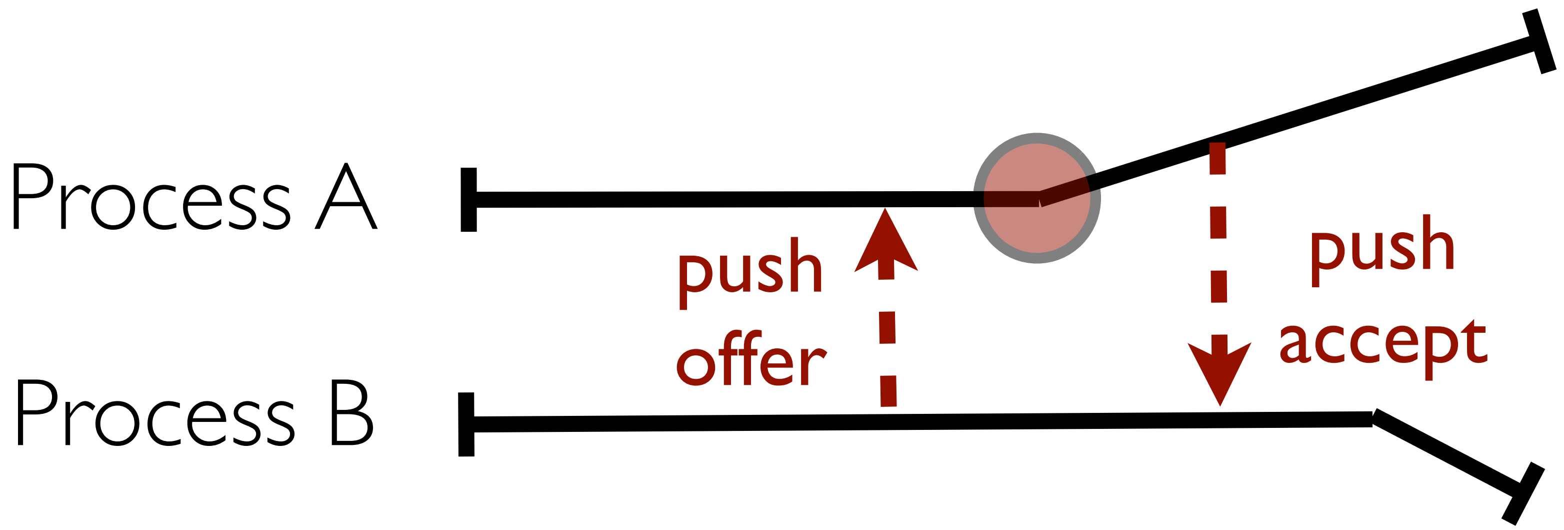
Solution:

Make thread's specification a *shareable resource*

Problem:

Cooperation is inherently non-thread-local

e.g. elimination stacks



Solution:

Make thread's specification a *shareable resource*

Algorithm	Local protocols	Tokens	Computational resources	Speculation	
Treiber's Stack	X				✓
Michael-Scott Queue	X				✓
Hand-over-hand set	X	X			✶
Elimination Flags	X	X	X		✓
Lazy set	X	X	X	X	✶
Conditional CAS	X	X	X	X	✓
K-CAS	X	X	X	X	✶

The Takeaway

Analyze fine-grained algorithms with fine-grained protocols!

To scale up, must address:
- varying roles
- cooperation
- nondeterminism

Our contributions:
- tokens
- computational resources
- speculation